INSTRUCTOR'S SOLUTIONS MANUAL

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ALGEBRA AND TRIGONOMETRY

SIXTH EDITION

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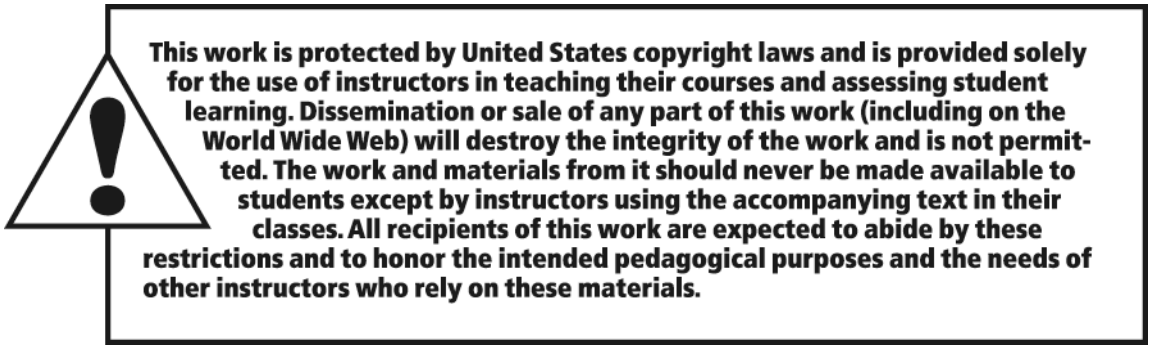
Pearson



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Chapter P
Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. **a.** Since 2014 is 14 years after 2000, substitute 14 for x.
$T = 4x^2+330x+3310$
$= 4(14)^2+330(14)+3310$
$= 8714$
The average cost of tuition and fees at public U.S. colleges for the school year ending in 2014 was \$8714.

b. The formula underestimates the actual answer by \$179.

3. The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\}\cap\{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\}\cup\{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9}=3$

b. Whole numbers: $0,\ \sqrt{9}$

c. Integers: $-9,\ 0,\ \sqrt{9}$

d. Rational numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

f. Real numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}$

6. **a.** $\left|1-\sqrt{2}\right|$
Because $\sqrt{2}\approx 1.4,$ the number inside the absolute value bars is negative. The absolute value of x when $x<0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi-3|$
Because $\pi\approx 3.14,$ the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi-3| = \pi-3.$

c. $\frac{|x|}{x}$
Because $x>0,\ \ |x| = x.$
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Concept and Vocabulary Check P.1

1. expression

2. b to the nth power; base; exponent

3. formula; modeling; models

4. intersection; $A\cap B$

5. union; $A\cup B$

6. natural

7. whole

8. integers

9. rational

10. irrational

11. rational; irrational

12. absolute value; x, $-x$

13. $b+a$; ba

14. $a+(b+c)$; $(ab)c$

15. $ab+ac$

16. 0; inverse; 0; identity

17. inverse; 1; identity

18. simplified

19. a

Exercise Set P.1

1. $7+5(10)=7+50=57$

2. $8+6(5)=8+30=38$

3. $6(3)-8=18-8=10$

4. $8(3)-4=24-4=20$

5. $8^2+3(8)=64+24=88$

6. $6^2+5(6)=36+30=66$

7. $7^2-6(7)+3=49-42+3=7+3=10$

8. $8^2-7(8)+4=64-56+4=8+4=12$

9. $4+5(9-7)^3=4+5(2)^3$
$=4+5(8)=4+40=44$

10. $6+5(8-6)^3=6+5(2)^3$
$=6+5(8)$
$=6+40=46$

11. $8^2-3(8-2)=64-3(6)$
$=64-18=46$

12. $8^2-4(8-3)=64-4(5)=64-20=44$

13. $\dfrac{5(x+2)}{2x-14}=\dfrac{5(10+2)}{2(10)-14}$
$=\dfrac{5(12)}{6}$
$=5\cdot 2$
$=10$

14. $\dfrac{7(x-3)}{2x-16}=\dfrac{7(9-3)}{2(9)-16}=\dfrac{7(6)}{2}=7\cdot 3=21$

15. $\dfrac{2x+3y}{x+1}; x=-2, y=4$
$=\dfrac{2(-2)+3(4)}{-2+1}=\dfrac{-4+12}{-1}=\dfrac{8}{-1}=-8$

16. $\dfrac{2x+y}{xy-2x}$; $x=-2$ and $y=4$
$\dfrac{2(-2)+4}{(-2)(4)-2(-2)}=\dfrac{-4+4}{-8+4}=\dfrac{0}{4}=0$

17. $C=\dfrac{5}{9}(50-32)=\dfrac{5}{9}(18)=10$
50°F is equivalent to 10°C.

18. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(86-32)=\dfrac{5}{9}(54)=30$
86°F is equivalent to 30°C.

19. $h=4+60t-16t^2=4+60(2)-16(2)^2$
$=4+120-16(4)=4+120-64$
$=124-64=60$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h = 4 + 60t - 16t^2$

$= 4 + 60(3) - 16(3)^2$

$= 4 + 180 - 16(9)$

$= 4 + 180 - 144$

$= 184 - 144 = 40$

Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1,2,3,4\} \cap \{2,4,5\} = \{2,4\}$

22. $\{1,3,7\} \cap \{2,3,8\} = \{3\}$

23. $\{s,e,t\} \cap \{t,e,s\} = \{s,e,t\}$

24. $\{r,e,a,l\} \cap \{l,e,a,r\} = \{r,e,a,l\}$

25. $\{1,3,5,7\} \cap \{2,4,6,8,10\} = \{\ \}$

The empty set is also denoted by $\varnothing$.

26. $\{1,3,5,7\} \cap \{-5,-3,-1\} = \{\ \}$ or $\varnothing$

27. $\{a,b,c,d\} \cap \varnothing = \varnothing$

28. $\{w,y,z\} \cap \varnothing = \varnothing$

29. $\{1,2,3,4\} \cup \{2,4,5\} = \{1,2,3,4,5\}$

30. $\{1,3,7,8\} \cup \{2,3,8\} = \{1,2,3,7,8\}$

31. $\{1,3,5,7\} \cup \{2,4,6,8,10\}$

$= \{1,2,3,4,5,6,7,8,10\}$

32. $\{0,1,3,5\} \cup \{2,4,6\} = \{0,1,2,3,4,5,6\}$

33. $\{a,e,i,o,u\} \cup \varnothing = \{a,e,i,o,u\}$

34. $\{e,m,p,t,y\} \cup \varnothing = \{e,m,p,t,y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. **a.** $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. **a.** $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. **a.** $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers will vary. An example is $\frac{1}{2}$.

41. Answers will vary. An example is 2.

42. Answers will vary. An example is -2.

43. true; -13 is to the left of -2 on the number line.

44. false; -6 is to the left of 2 on the number line.

45. true; 4 is to the right of -7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi = -\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300| = 300$

52. $|-203| = 203$

53. $|12 - \pi| = 12 - \pi$

54. $|7 - \pi| = 7 - \pi$

55. $|\sqrt{2} - 5| = 5 - \sqrt{2}$

56. $|\sqrt{5} - 13| = 13 - \sqrt{5}$

57. $\dfrac{-3}{|-3|} = \dfrac{-3}{3} = -1$

58. $\dfrac{-7}{|-7|} = \dfrac{-7}{7} = -1$

59. $\big||-3| - |-7|\big| = |3 - 7| = |-4| = 4$

60. $\big||-5| - |-13|\big| = |5 - 13| = |-8| = 8$

61. $|x + y| = |2 + (-5)| = |-3| = 3$

62. $|x - y| = |2 - (-5)| = |7| = 7$

63. $|x| + |y| = |2| + |-5| = 2 + 5 = 7$

64. $|x| - |y| = |2| - |-5| = 2 - 5 = -3$

65. $\dfrac{y}{|y|} = \dfrac{-5}{|-5|} = \dfrac{-5}{5} = -1$

66. $\dfrac{|x|}{x} + \dfrac{|y|}{y} = \dfrac{|2|}{2} + \dfrac{|-5|}{-5} = \dfrac{2}{2} + \dfrac{5}{-5} = 1 + (-1) = 0$

67. The distance is $|2 - 17| = |-15| = 15$.

68. The distance is $|4 - 15| = |-11| = 11$.

69. The distance is $|-2 - 5| = |-7| = 7$.

70. The distance is $|-6 - 8| = |-14| = 14$.

71. The distance is $|-19 - (-4)| = |-19 + 4| = |-15| = 15$.

72. The distance is $|-26 - (-3)| = |-26 + 3| = |-23| = 23$.

73. The distance is
$|-3.6 - (-1.4)| = |-3.6 + 1.4| = |-2.2| = 2.2$.

74. The distance is
$|-5.4 - (-1.2)| = |-5.4 + 1.2| = |-4.2| = 4.2$.

75. $6 + (-4) = (-4) + 6$;
commutative property of addition

76. $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$;
distributive property of multiplication over addition

77. $6 + (2 + 7) = (6 + 2) + 7$;
associative property of addition

78. $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$;
commutative property of multiplication

79. $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$;
commutative property of addition

80. $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$;
commutative property of multiplication

81. $2(-8 + 6) = -16 + 12$;
distributive property of multiplication over addition

82. $-8(3 + 11) = -24 + (-88)$;
distributive property of multiplication over addition

83. $\dfrac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4) + \big[-(x+4)\big] = 0$;
inverse property of addition

85. $\begin{aligned} 5(3x+4) - 4 &= 5 \cdot 3x + 5 \cdot 4 - 4 \\ &= 15x + 20 - 4 \\ &= 15x + 16 \end{aligned}$

86. $2(5x+4)-3 = 2\cdot 5x+2\cdot 4-3$
$= 10x+8-3$
$= 10x+5$

87. $5(3x-2)+12x = 5\cdot 3x-5\cdot 2+12x$
$= 15x-10+12x$
$= 27x-10$

88. $2(5x-1)+14x = 2\cdot 5x-2\cdot 1+14x$
$= 10x-2+14x$
$= 24x-2$

89. $7(3y-5)+2(4y+3)$
$= 7\cdot 3y-7\cdot 5+2\cdot 4y+2\cdot 3$
$= 21y-35+8y+6$
$= 29y-29$

90. $4(2y-6)+3(5y+10)$
$= 4\cdot 2y-4\cdot 6+3\cdot 5y+3\cdot 10$
$= 8y-24+15y+30$
$= 23y+6$

91. $5(3y-2)-(7y+2) = 15y-10-7y-2$
$= 8y-12$

92. $4(5y-3)-(6y+3) = 20y-12-6y-3$
$= 14y-15$

93. $7-4\left[3-(4y-5)\right] = 7-4[3-4y+5]$
$= 7-4[8-4y]$
$= 7-32+16y$
$= 16y-25$

94. $6-5[8-(2y-4)] = 6-5[8-2y+4]$
$= 6-5[12-2y]$
$= 6-60+10y$
$= 10y-54$

95. $18x^2+4-\left[6\left(x^2-2\right)+5\right]$
$= 18x^2+4-\left[6x^2-12+5\right]$
$= 18x^2+4-\left[6x^2-7\right]$
$= 18x^2+4-6x^2+7$
$= 18x^2-6x^2+4+7$
$= (18-6)x^2+11 = 12x^2+11$

96. $14x^2+5-\left[7\left(x^2-2\right)+4\right]$
$= 14x^2+5-\left[7x^2-14+4\right]$
$= 14x^2+5-\left[7x^2-10\right]$
$= 14x^2+5-7x^2+10$
$= 14x^2-7x^2+5+10$
$= (14-7)x^2+15$
$= 7x^2+15$

97. $-(-14x) = 14x$

98. $-(-17y) = 17y$

99. $-(2x-3y-6) = -2x+3y+6$

100. $-(5x-13y-1) = -5x+13y+1$

101. $\frac{1}{3}(3x)+[(4y)+(-4y)] = x+0 = x$

102. $\frac{1}{2}(2y)+[(-7x)+7x] = y+0 = y$

103. $|-6| \square |-3|$
$6 \square 3$
$6 > 3$
Since $6>3$, $|-6|>|-3|$.

104. $|-20| \square |-50|$
$20 \square 50$
$20 < 50$
Since $20<50$, $|-20|<|-50|$.

105. $\left|\frac{3}{5}\right| \square |-0.6|$
$|0.6| \square |-0.6|$
$0.6 \square 0.6$
$0.6 = 0.6$
Since $0.6=0.6$, $\left|\frac{3}{5}\right| = |-0.6|$.

106. $\left|\frac{5}{2}\right| \square |-2.5|$

$|2.5| \square |-2.5|$

$2.5 \square 2.5$

$2.5 = 2.5$

Since $2.5 = 2.5$, $\left|\frac{5}{2}\right| = |-2.5|$.

107. $\frac{30}{40} - \frac{3}{4} \square \frac{14}{15} \cdot \frac{15}{14}$

$\frac{30}{40} - \frac{30}{40} \square \frac{\cancel{14}}{\cancel{15}} \cdot \frac{\cancel{15}}{\cancel{14}}$

$0 \square 1$

$0 < 1$

Since $0 < 1$, $\frac{30}{40} - \frac{3}{4} < \frac{14}{15} \cdot \frac{15}{14}$.

108. $\frac{17}{18} \cdot \frac{18}{17} \square \frac{50}{60} - \frac{5}{6}$

$\frac{\cancel{17}}{\cancel{18}} \cdot \frac{\cancel{18}}{\cancel{17}} \square \frac{50}{60} - \frac{50}{60}$

$1 \square 0$

$1 > 0$

Since $1 > 0$, $\frac{17}{18} \cdot \frac{18}{17} > \frac{50}{60} - \frac{5}{6}$.

109. $\frac{8}{13} \div \frac{8}{13} \square |-1|$

$\frac{8}{13} \cdot \frac{13}{8} \square 1$

$1 \square 1$

$1 = 1$

Since $1 = 1$, $\frac{8}{13} \div \frac{8}{13} = |-1|$.

110. $|-2| \square \frac{4}{17} \div \frac{4}{17}$

$2 \square \frac{4}{17} \cdot \frac{17}{4}$

$2 \square 1$

$2 > 1$

Since $2 > 1$, $|-2| > \frac{4}{17} \div \frac{4}{17}$.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3 = 64 - 16 \div 4 \cdot 4 - 3$

$= 64 - 4 \cdot 4 - 3$

$= 64 - 16 - 3$

$= 48 - 3$

$= 45$

112. $10^2 - 100 \div 5^2 \cdot 2 - 3 = 100 - 100 \div 25 \cdot 2 - 3$

$= 100 - 4 \cdot 2 - 3$

$= 100 - 8 - 3$

$= 92 - 3$

$= 89$

113. $\frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2} = \frac{5 \cdot 2 - 9}{[9 - (-2)]^2}$

$= \frac{10 - 9}{[9 + 2]^2}$

$= \frac{10 - 9}{11^2}$

$= \frac{1}{121}$

114. $\frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2} = \frac{5 + 12}{(12 - 6)^2}$

$= \frac{17}{6^2}$

$= \frac{17}{36}$

115. $8 - 3[-2(2 - 5) - 4(8 - 6)] = 8 - 3[-2(-3) - 4(2)]$

$= 8 - 3[6 - 8]$

$= 8 - 3[-2]$

$= 8 + 6$

$= 14$

116. $8 - 3[-2(5 - 7) - 5(4 - 2)] = 8 - 3[-2(-2) - 5(2)]$

$= 8 - 3[4 - 10]$

$= 8 - 3[-6]$

$= 8 + 18$

$= 26$

117. $\frac{2(-2) - 4(-3)}{5 - 8} = \frac{-4 + 12}{-3}$

$= \frac{8}{-3}$

$= -\frac{8}{3}$

118. $\dfrac{6(-4)-5(-3)}{9-10} = \dfrac{-24+15}{-1}$

$= \dfrac{-9}{-1}$

$= 9$

119. $\dfrac{(5-6)^2-2\left|3-7\right|}{89-3\cdot 5^2} = \dfrac{(-1)^2-2\left|-4\right|}{89-3\cdot 25}$

$= \dfrac{1-2(4)}{89-75}$

$= \dfrac{1-8}{14}$

$= \dfrac{-7}{14}$

$= -\dfrac{1}{2}$

120. $\dfrac{12\div 3\cdot 5\left|2^2+3^2\right|}{7+3-6^2} = \dfrac{12\div 3\cdot 5\left|4+9\right|}{7+3-36}$

$= \dfrac{4\cdot 5\left|13\right|}{10-36}$

$= \dfrac{20(13)}{-26}$

$= \dfrac{260}{-26}$

$= -10$

121. $x-(x+4) = x-x-4 = -4$

122. $x-(8-x) = x-8+x = 2x-8$

123. $6(-5x) = -30x$

124. $10(-4x) = -40x$

125. $5x-2x = 3x$

126. $6x-(-2x) = 6x+2x = 8x$

127. $8x-(3x+6) = 8x-3x-6 = 5x-6$

128. $8-3(x+6) = 8-3x-18 = -3x-10$

129. **a.** $H = \dfrac{7}{10}(220-a)$

$H = \dfrac{7}{10}(220-20)$

$= \dfrac{7}{10}(200)$

$= 140$

The lower limit of the heart rate for a 20-year-old with this exercise goal is 140 beats per minute.

b. $H = \dfrac{4}{5}(220-a)$

$H = \dfrac{4}{5}(220-20)$

$= \dfrac{4}{5}(200)$

$= 160$

The upper limit of the heart rate for a 20-year-old with this exercise goal is 160 beats per minute.

130. **a.** $H = \dfrac{1}{2}(220-a)$

$H = \dfrac{1}{2}(220-30)$

$= \dfrac{1}{2}(190)$

$= 95$

The lower limit of the heart rate for a 30-year-old with this exercise goal is 95 beats per minute.

b. $H = \dfrac{3}{5}(220-a)$

$H = \dfrac{3}{5}(220-30)$

$= \dfrac{3}{5}(190)$

$= 114$

The upper limit of the heart rate for a 30-year-old with this exercise goal is 114 beats per minute.

131. **a.** $T = 21x^2+862x+15{,}552$

$= 21(14)^2+862(14)+15{,}552$

$= 31{,}736$

The formula estimates the cost to have been \$31,736 in 2014.

b. This overestimates the value in the graph by \$35.

c. $T = 21x^2 + 862x + 15{,}552$
$= 21(20)^2 + 862(20) + 15{,}552$
$= 41{,}192$
The formula projects the cost to be \$41,192 in 2020.

132. a. $T = 21x^2 + 862x + 15{,}552$
$= 21(12)^2 + 862(12) + 15{,}552$
$= 28{,}920$
The formula estimates the cost to have been \$28,920 in 2012.

b. This underestimates the value in the graph by \$136.

c. $T = 21x^2 + 862x + 15{,}552$
$= 21(22)^2 + 862(22) + 15{,}552$
$= 44{,}680$
The formula projects the cost to be \$44,680 in 2022.

133. a. $0.05x + 0.12(10{,}000 - x)$
$= 0.05x + 1200 - 0.12x$
$= 1200 - 0.07x$

b. $1200 - 0.07x = 1200 - 0.07(6000)$
$= \$780$

134. a. $0.06t + 0.5(50 - t) = 0.06t + 25 - 0.5t$
$= 25 - 0.44t$

b. $0.06(20) + 0.5(50 - 20)$
$= 1.2 + 0.5(30)$
$= 1.2 + 15$
$= 16.2$ miles

135. – 143. Answers will vary.

144. does not make sense; Explanations will vary. Sample explanation: Models do not always accurately predict future values.

145. does not make sense; Explanations will vary. Sample explanation: To use the model, substitute 0 for x.

146. makes sense

147. does not make sense; Explanations will vary. Sample explanation: The commutative property changes order and the associative property changes groupings.

148. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.

149. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers.

150. true

151. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.

152. false; Changes to make the statement true will vary. A sample change is: The term x has a coefficient of 1.

153. false; Changes to make the statement true will vary. A sample change is:
$5 + 3(x - 4) = 5 + 3x - 12 = 3x - 7.$

154. false; Changes to make the statement true will vary. A sample change is: $-x - x = -2x.$

155. true

156. $\sqrt{2} \approx 1.4$
$1.4 < 1.5$
$\sqrt{2} < 1.5$

157. $-\pi > -3.5$

158. $-\frac{3.14}{2} = -1.57$
$-\frac{\pi}{2} \approx -1.571$
$-1.57 > -1.571$
$-\frac{3.14}{2} > -\frac{\pi}{2}$

159. a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^7$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^{10}$

c. add the exponents

160. a. $\dfrac{b^7}{b^3} = \dfrac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^4$

b. $\dfrac{b^8}{b^2} = \dfrac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^6$

c. subtract the exponents

161. $6.2 \times 10^3 = 6.2 \times 10 \times 10 \times 10 = 6200$
It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. a. $3^3 \cdot 3^2 = 3^{3+2} = 3^5$ or 243

b. $\left(4x^3y^4\right)\left(10x^2y^6\right) = 4 \cdot 10 \cdot x^3 \cdot x^2 \cdot y^4 \cdot y^6$
$= 40x^{3+2} \cdot y^{4+6}$
$= 40x^5y^{10}$

2. a. $\dfrac{(-3)^6}{(-3)^3} = (-3)^3 = -27$

b. $\dfrac{27x^{14}y^8}{3x^3y^5} = \dfrac{27}{3} \cdot \dfrac{x^{14}}{x^3} \cdot \dfrac{y^8}{y^5} = 9x^{14-3}y^{8-5} = 9x^{11}y^3$

3. a. $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

b. $(-3)^{-3} = \dfrac{1}{(-3)^3} = \dfrac{1}{-27} = -\dfrac{1}{27}$

c. $\dfrac{1}{4^{-2}} = \dfrac{1}{\frac{1}{4^2}} = 1 \cdot \dfrac{4^2}{1} = 4^2 = 16$

d. $3x^{-6}y^4 = 3 \cdot \dfrac{1}{x^6} \cdot y^4 = \dfrac{3y^4}{x^6}$

4. a. $\left(3^3\right)^2 = 3^{3 \cdot 2} = 3^6$ or 729

b. $\left(y^7\right)^{-2} = y^{7(-2)} = y^{-14} = \dfrac{1}{y^{14}}$

c. $\left(b^{-3}\right)^{-4} = b^{-3(-4)} = b^{12}$

5. $(-4x)^3 = (-4)^3(x)^3 = -64x^3$

6. a. $\left(-\dfrac{2}{y}\right)^5 = \dfrac{(-2)^5}{y^5} = \dfrac{-32}{y^5}$

b. $\left(\dfrac{x^5}{3}\right)^3 = \dfrac{\left(x^5\right)^3}{3^3} = \dfrac{x^{15}}{27}$

7. a. $\left(2x^3y^6\right)^4 = (2)^4\left(x^3\right)^4\left(y^6\right)^4 = 16x^{12}y^{24}$

b. $\left(-6x^2y^5\right)\left(3xy^3\right) = (-6) \cdot 3 \cdot x^2 \cdot x \cdot y^5 \cdot y^3$
$= -18x^3y^8$

c. $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}} = \left(\dfrac{100}{20}\right)\left(\dfrac{x^{12}}{x^{16}}\right)\left(\dfrac{y^2}{y^{-4}}\right)$
$= 5x^{12-16}y^{2-(-4)}$
$= 5x^{-4}y^6$
$= \dfrac{5y^6}{x^4}$

d. $\left(\dfrac{5x}{y^4}\right)^{-2} = \dfrac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$= \dfrac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$= \dfrac{5^{-2}x^{-2}}{y^{-8}}$
$= \dfrac{y^8}{5^2x^2}$
$= \dfrac{y^8}{25x^2}$

8. a. $-2.6 \times 10^9 = -2,600,000,000$

b. $3.017 \times 10^{-6} = 0.000003017$

9. a. $5,210,000,000 = 5.21 \times 10^9$

b. $-0.00000006893 = -6.893 \times 10^{-8}$

10. $410\times10^7 = \left(4.1\times10^2\right)\times10^7$
$= 4.1\times\left(10^2\times10^7\right)$
$= 4.1\times10^9$

11 a. $\left(7.1\times10^5\right)\left(5\times10^{-7}\right)$
$= 7.1\cdot5\times10^5\cdot10^{-7}$
$= 35.5\times10^{-2}$
$= \left(3.55\times10^1\right)\times10^{-2}$
$= 3.55\times\left(10^1\times10^{-2}\right)$
$= 3.55\times10^{-1}$

b. $\dfrac{1.2\times10^6}{3\times10^{-3}} = \dfrac{1.2}{3}\cdot\dfrac{10^6}{10^{-3}}$
$= 0.4\times10^{6-(-3)}$
$= 0.4\times10^9$
$= 4\times10^8$

12. $\dfrac{4.08\times10^{10}}{680,000} = \dfrac{4.08\times10^{10}}{6.8\times10^5} = \dfrac{4.08}{6.8}\cdot\dfrac{10^{10}}{10^5}$
$= 0.6\times10^5$
$= 60,000$
The average salary was \$60,000 per U.S. police officer.

Concept and Vocabulary Check P.2

1. b^{m+n}; add

2. b^{m-n}; subtract

3. 1

4. $\dfrac{1}{b^n}$

5. false

6. b^n

7. true

8. a number greater than or equal to 1 and less than 10; integer

9. true

10. false

Exercise Set P.2

1. $5^2\cdot2 = (5\cdot5)\cdot2 = 25\cdot2 = 50$

2. $6^2\cdot2 = (6\cdot6)\cdot2 = 36\cdot2 = 72$

3. $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

4. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

5. $-2^6 = -2\cdot2\cdot2\cdot2\cdot2\cdot2 = -64$

6. $-2^4 = -2\cdot2\cdot2\cdot2 = -16$

7. $(-3)^0 = 1$

8. $(-9)^0 = 1$

9. $-3^0 = -1$

10. $-9^0 = -1$

11. $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{4\cdot4\cdot4} = \dfrac{1}{64}$

12. $2^{-6} = \dfrac{1}{2^6} = \dfrac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2} = \dfrac{1}{64}$

13. $2^2\cdot2^3 = 2^{2+3} = 2^5 = 2\cdot2\cdot2\cdot2\cdot2 = 32$

14. $3^3\cdot3^2 = 3^{3+2} = 3^5 = 3\cdot3\cdot3\cdot3\cdot3 = 243$

15. $(2^2)^3 = 2^{2\cdot3} = 2^6 = 2\cdot2\cdot2\cdot2\cdot2\cdot2 = 64$

16. $(3^3)^2 = 3^{3\cdot2} = 3^6 = 3\cdot3\cdot3\cdot3\cdot3\cdot3 = 729$

17. $\dfrac{2^8}{2^4} = 2^{8-4} = 2^4 = 2\cdot2\cdot2\cdot2 = 16$

18. $\dfrac{3^8}{3^4} = 3^{8-4} = 3^4 = 3\cdot3\cdot3\cdot3 = 81$

19. $3^{-3}\cdot3 = 3^{-3+1} = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{3\cdot3} = \dfrac{1}{9}$

20. $2^{-3}\cdot2 = 2^{-3+1} = 2^{-2} = \dfrac{1}{2^2} = \dfrac{1}{2\cdot2} = \dfrac{1}{4}$

21. $\dfrac{2^3}{2^7} = 2^{3-7} = 2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{2\cdot2\cdot2\cdot2} = \dfrac{1}{16}$

22. $\dfrac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \dfrac{1}{3^3} = \dfrac{1}{3\cdot 3\cdot 3} = \dfrac{1}{27}$

23. $x^{-2}y = \dfrac{1}{x^2}\cdot y = \dfrac{y}{x^2}$

24. $xy^{-3} = x\cdot\dfrac{1}{y^3} = \dfrac{x}{y^3}$

25. $x^0y^5 = 1\cdot y^5 = y^5$

26. $x^7\cdot y^0 = x^7\cdot 1 = x^7$

27. $x^3\cdot x^7 = x^{3+7} = x^{10}$

28. $x^{11}\cdot x^5 = x^{11+5} = x^{16}$

29. $x^{-5}\cdot x^{10} = x^{-5+10} = x^5$

30. $x^{-6}\cdot x^{12} = x^{-6+12} = x^6$

31. $(x^3)^7 = x^{3\cdot 7} = x^{21}$

32. $(x^{11})^5 = x^{11\cdot 5} = x^{55}$

33. $(x^{-5})^3 = x^{-5\cdot 3} = x^{-15} = \dfrac{1}{x^{15}}$

34. $(x^{-6})^4 = x^{-6\cdot 4} = x^{-24} = \dfrac{1}{x^{24}}$

35. $\dfrac{x^{14}}{x^7} = x^{14-7} = x^7$

36. $\dfrac{x^{30}}{x^{10}} = x^{30-10} = x^{20}$

37. $\dfrac{x^{14}}{x^{-7}} = x^{14-(-7)} = x^{14+7} = x^{21}$

38. $\dfrac{x^{30}}{x^{-10}} = x^{30-(-10)} = x^{30+10} = x^{40}$

39. $(8x^3)^2 = 8^2(x^3)^2 = 8^2x^{3\cdot 2} = 64x^6$

40. $(6x^4)^2 = (6)^2(x^4)^2 = 6^2x^{4\cdot 2} = 36x^8$

41. $\left(-\dfrac{4}{x}\right)^3 = \dfrac{(-4)^3}{x^3} = -\dfrac{64}{x^3}$

42. $\left(-\dfrac{6}{y}\right)^3 = \dfrac{(-6)^3}{y^3} = -\dfrac{216}{y^3}$

43. $\begin{aligned}(-3x^2y^5)^2 &= (-3)^2(x^2)^2\cdot(y^5)^2\\ &= 9x^{2\cdot 2}y^{5\cdot 2}\\ &= 9x^4y^{10}\end{aligned}$

44. $\begin{aligned}(-3x^4y^6)^3 &= (-3)^3(x^4)^3(y^6)^3\\ &= -27x^{4\cdot 3}y^{6\cdot 3}\\ &= -27x^{12}y^{18}\end{aligned}$

45. $(3x^4)(2x^7) = 3\cdot 2x^4\cdot x^7 = 6x^{4+7} = 6x^{11}$

46. $(11x^5)(9x^{12}) = 11\cdot 9x^5x^{12} = 99x^{5+12} = 99x^{17}$

47. $\begin{aligned}(-9x^3y)(-2x^6y^4) &= (-9)(-2)x^3x^6yy^4\\ &= 18x^{3+6}y^{1+4}\\ &= 18x^9y^5\end{aligned}$

48. $\begin{aligned}(-5x^4y)(-6x^7y^{11}) &= (-5)(-6)x^4x^7yy^{11}\\ &= 30x^{4+7}y^{1+11}\\ &= 30x^{11}y^{12}\end{aligned}$

49. $\dfrac{8x^{20}}{2x^4} = \left(\dfrac{8}{2}\right)\left(\dfrac{x^{20}}{x^4}\right) = 4x^{20-4} = 4x^{16}$

50. $\dfrac{20x^{24}}{10x^6} = \left(\dfrac{20}{10}\right)\left(\dfrac{x^{24}}{x^6}\right) = 2x^{24-6} = 2x^{18}$

51. $\begin{aligned}\dfrac{25a^{13}\cdot b^4}{-5a^2\cdot b^3} &= \left(\dfrac{25}{-5}\right)\left(\dfrac{a^{13}}{a^2}\right)\left(\dfrac{b^4}{b^3}\right)\\ &= -5a^{13-2}b^{4-3}\\ &= -5a^{11}b\end{aligned}$

52. $\begin{aligned}\dfrac{35a^{14}b^6}{-7a^7b^3} &= \left(\dfrac{35}{-7}\right)\left(\dfrac{a^{14}}{a^7}\right)\left(\dfrac{b^6}{b^3}\right)\\ &= -5a^{14-7}b^{6-3}\\ &= -5a^7b^3\end{aligned}$

53. $\dfrac{14b^7}{7b^{14}} = \left(\dfrac{14}{7}\right)\left(\dfrac{b^7}{b^{14}}\right) = 2\cdot b^{7-14} = 2b^{-7} = \dfrac{2}{b^7}$

54. $\dfrac{20b^{10}}{10b^{20}} = \left(\dfrac{20}{10}\right)\left(\dfrac{b^{10}}{b^{20}}\right)$

$= 2b^{10-20}$

$= 2b^{-10}$

$= \dfrac{2}{b^{10}}$

55. $(4x^3)^{-2} = (4^{-2})(x^3)^{-2}$

$= 4^{-2}x^{-6}$

$= \dfrac{1}{4^2 x^6}$

$= \dfrac{1}{16x^6}$

56. $(10x^2)^{-3} = 10^{-3}x^{2\cdot(-3)}$

$= 10^{-3}x^{-6}$

$= \dfrac{1}{10^3 x^6}$

$= \dfrac{1}{1000x^6}$

57. $\dfrac{24x^3 \cdot y^5}{32x^7 y^{-9}} = \dfrac{3}{4}x^{3-7}y^{5-(-9)}$

$= \dfrac{3}{4}x^{-4}y^{14}$

$= \dfrac{3y^{14}}{4x^4}$

58. $\dfrac{10x^4 y^9}{30x^{12}y^{-3}} = \dfrac{1}{3}x^{4-12}y^{9-(-3)}$

$= \dfrac{1}{3}x^{-8}y^{12}$

$= \dfrac{y^{12}}{3x^8}$

59. $\left(\dfrac{5x^3}{y}\right)^{-2} = \dfrac{5^{-2}x^{-6}}{y^{-2}} = \dfrac{y^2}{25x^6}$

60. $\left(\dfrac{3x^4}{y}\right)^{-3} = \left(\dfrac{y}{3x^4}\right)^3$

$= \dfrac{y^3}{3^3 x^{4\cdot 3}}$

$= \dfrac{y^3}{27x^{12}}$

61. $\left(\dfrac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3 = \left(\dfrac{-3b^{2-(-3)}}{a^{10-4}}\right)^3$

$= \left(\dfrac{-3b^5}{a^6}\right)^3$

$= \dfrac{-27b^{15}}{a^{18}}$

62. $\left(\dfrac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3 = \left(\dfrac{-3b^{8-(-2)}}{a^{17-14}}\right)^3$

$= \left(\dfrac{-3b^{10}}{a^3}\right)^3$

$= \dfrac{-27b^{30}}{a^9}$

63. $\left(\dfrac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0 = 1$

64. $\left(\dfrac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0 = 1$

65. $3.8\times10^2 = 380$

66. $9.2\times10^2 = 920$

67. $6\times10^{-4} = 0.0006$

68. $7\times10^{-5} = 0.00007$

69. $-7.16\times10^6 = -7{,}160{,}000$

70. $-8.17\times10^6 = -8{,}170{,}000$

71. $7.9\times10^{-1} = 0.79$

72. $6.8\times10^{-1} = 0.68$

73. $-4.15\times10^{-3} = -0.00415$

74. $-3.14\times10^{-3} = -0.00314$

75. $-6.00001\times10^{10} = -60,000,100,000$

76. $-7.00001\times10^{10} = -70,000,100,000$

77. $32,000 = 3.2\times10^{4}$

78. $64,000 = 6.4\times10^{4}$

79. $638,000,000,000,000,000$
$= 6.38\times10^{17}$

80. $579,000,000,000,000,000 = 5.79\times10^{17}$

81. $-5716 = -5.716\times10^{3}$

82. $-3829 = -3.829\times10^{3}$

83. $0.0027 = 2.7\times10^{-3}$

84. $0.0083 = 8.3\times10^{-3}$

85. $-0.00000000504 = -5.04\times10^{-9}$

86. $-0.00000000405 = -4.05\times10^{-9}$

87. $\left(3\times10^{4}\right)\left(2.1\times10^{3}\right) = (3\times2.1)\left(10^{4}\times10^{3}\right)$
$= 6.3\times10^{4+3} = 6.3\times10^{7}$

88. $\left(2\times10^{4}\right)\left(4.1\times10^{3}\right) = 8.2\times10^{7}$

89. $\left(1.6\times10^{15}\right)\left(4\times10^{-11}\right) = (1.6\times4)\left(10^{15}\times10^{-11}\right)$
$= 6.4\times10^{15+(-11)}$
$= 6.4\times10^{4}$

90. $\left(1.4\times10^{15}\right)\left(3\times10^{-11}\right) = (1.4\times3)\left(10^{15}\times10^{-11}\right)$
$= 4.2\times10^{15+(-11)}$
$= 4.2\times10^{4}$

91. $\left(6.1\times10^{-8}\right)\left(2\times10^{-4}\right) = (6.1\times2)\left(10^{-8}\times10^{-4}\right)$
$= 12.2\times10^{-8+(-4)}$
$= 12.2\times10^{-12}$
$= 1.22\times10^{-11}$

92. $\left(5.1\times10^{-8}\right)\left(3\times10^{-4}\right) = 15.3\times10^{-12}$
$= 1.53\times10^{-11}$

93. $\left(4.3\times10^{8}\right)\left(6.2\times10^{4}\right)$
$= (4.3\times6.2)\left(10^{8}\times10^{4}\right)$
$= 26.66\times10^{8+4}$
$= 26.66\times10^{12}$
$= 2.666\times10^{13} \approx 2.67\times10^{13}$

94. $\left(8.2\times10^{8}\right)\left(4.6\times10^{4}\right)$
$= 37.72\times10^{8+4} = 37.72\times10^{12}$
$= 3.772\times10^{13} \approx 3.77\times10^{13}$

95. $\dfrac{8.4\times10^{8}}{4\times10^{5}} = \dfrac{8.4}{4}\times\dfrac{10^{8}}{10^{5}}$
$= 2.1\times10^{8-5} = 2.1\times10^{3}$

96. $\dfrac{6.9\times10^{8}}{3\times10^{5}} = 2.3\times10^{8-5} = 2.3\times10^{3}$

97. $\dfrac{3.6\times10^{4}}{9\times10^{-2}} = \dfrac{3.6}{9}\times\dfrac{10^{4}}{10^{-2}}$
$= 0.4\times10^{4-(-2)}$
$= 0.4\times10^{6} = 4\times10^{5}$

98. $\dfrac{1.2\times10^{4}}{2\times10^{-2}} = 0.6\times10^{4-(-2)} = 0.6\times10^{6}$
$= \left(6\times10^{-1}\right)\times10^{6} = 6\times10^{5}$

99. $\dfrac{4.8\times10^{-2}}{2.4\times10^{6}} = \dfrac{4.8}{2.4}\times\dfrac{10^{-2}}{10^{6}}$
$= 2\times10^{-2-6} = 2\times10^{-8}$

100. $\dfrac{7.5\times10^{-2}}{2.5\times10^{6}} = 3\times10^{-2-6} = 3\times10^{-8}$

101. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}} = \dfrac{2.4}{4.8}\times\dfrac{10^{-2}}{10^{-6}}$
$= 0.5\times10^{-2-(-6)}$
$= 0.5\times10^{4} = 5\times10^{3}$

102. $\dfrac{1.5\times10^{-2}}{5\times10^{-6}} = 0.5\times10^{-2-(-6)}$

$= 0.5\times10^{4} = 5\times10^{3}$

103. $\dfrac{480,000,000,000}{0.00012} = \dfrac{4.8\times10^{11}}{1.2\times10^{-4}}$

$= \dfrac{4.8}{1.2}\times\dfrac{10^{11}}{10^{-4}}$

$= 4\times10^{11-(-4)}$

$= 4\times10^{15}$

104. $\dfrac{282,000,000,000}{0.00141} = \dfrac{2.82\times10^{11}}{1.41\times10^{-3}}$

$= 2\times10^{11-(-3)}$

$= 2\times10^{14}$

105. $\dfrac{0.00072\times0.003}{0.00024}$

$= \dfrac{\left(7.2\times10^{-4}\right)\left(3\times10^{-3}\right)}{2.4\times10^{-4}}$

$= \dfrac{7.2\times3}{2.4}\times\dfrac{10^{-4}\cdot10^{-3}}{10^{-4}} = 9\times10^{-3}$

106. $\dfrac{66000\times0.001}{0.003\times0.002} = \dfrac{\left(6.6\times10^{4}\right)\left(1\times10^{-3}\right)}{\left(3\times10^{-3}\right)\left(2\times10^{-3}\right)}$

$= \dfrac{6.6\times10^{1}}{6\times10^{-6}} = 1.1\times10^{1-(-6)}$

$= 1.1\times10^{7}$

107. $\dfrac{\left(x^{-2}y\right)^{-3}}{\left(x^{2}y^{-1}\right)^{3}} = \dfrac{x^{6}y^{-3}}{x^{6}y^{-3}}$

$= x^{6-6}y^{-3-(-3)} = x^{0}y^{0} = 1$

108. $\dfrac{\left(xy^{-2}\right)^{-2}}{\left(x^{-2}y\right)^{-3}} = \dfrac{x^{-2}y^{4}}{x^{6}y^{-3}}$

$= x^{-2-6}y^{4-(-3)} = x^{-8}y^{7} = \dfrac{y^{7}}{x^{8}}$

109. $\left(2x^{-3}yz^{-6}\right)(2x)^{-5} = 2x^{-3}yz^{-6}\cdot2^{-5}x^{\cdot-5}$

$= 2^{-4}x^{-8}yz^{-6} = \dfrac{y}{2^{4}x^{8}z^{6}} = \dfrac{y}{16x^{8}z^{6}}$

110. $\left(3x^{-4}yz^{-7}\right)(3x)^{-3} = 3x^{-4}yz^{-7}\cdot3^{-3}x^{\cdot-3}$

$= 3^{-2}x^{-7}yz^{-7} = \dfrac{y}{3^{2}x^{7}z^{7}} = \dfrac{y}{9x^{7}z^{7}}$

111. $\left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2} = \left(x^{6}y^{8}z^{10}\right)^{-2}$

$= x^{-12}y^{-16}z^{-20} = \dfrac{1}{x^{12}y^{16}z^{20}}$

112. $\left(\dfrac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4} = \left(x^{8}y^{10}z^{12}\right)^{-4}$

$= x^{-32}y^{-40}z^{-48} = \dfrac{1}{x^{32}y^{40}z^{48}}$

113. $\dfrac{\left(2^{-1}x^{-2}y^{-1}\right)^{-2}\left(2x^{-4}y^{3}\right)^{-2}\left(16x^{-3}y^{3}\right)^{0}}{\left(2x^{-3}y^{-5}\right)^{2}}$

$= \dfrac{\left(2^{2}x^{2}y^{2}\right)\left(2^{-2}x^{8}y^{-6}\right)(1)}{\left(2^{2}x^{-6}y^{-10}\right)}$

$= \dfrac{x^{18}y^{6}}{4}$

114. $\dfrac{\left(2^{-1}x^{-3}y^{-1}\right)^{-2}\left(2x^{-6}y^{4}\right)^{-2}\left(9x^{3}y^{-3}\right)^{0}}{\left(2x^{-4}y^{-6}\right)^{2}}$

$= \dfrac{\left(2^{2}x^{6}y^{2}\right)\left(2^{-2}x^{12}y^{-8}\right)(1)}{\left(2^{2}x^{-8}y^{-12}\right)}$

$= \dfrac{x^{26}y^{6}}{4}$

115. **a.** 3.18×10^{12}

b. 3.20×10^{8}

c. $\dfrac{3.18\times10^{12}}{3.20\times10^{8}} = \dfrac{3.18}{3.20}\times\dfrac{10^{12}}{10^{8}}$

$\approx 0.9938\times10^{4}$

≈ 9938

\$9938 per American

116. **a.** 3.02×10^{12}

b. 3.19×10^{8}

c. $\dfrac{3.02\times10^{12}}{3.19\times10^{8}} = \dfrac{3.02}{3.19}\times\dfrac{10^{12}}{10^{8}}$
$\approx 0.9467\times10^{4}$
≈ 9467
\$9467 per American

117. a. 1.89×10^{13}

b. 6×10^{4}

c. $\dfrac{1.89\times10^{13}}{6\times10^{4}} = \dfrac{1.89}{6}\times\dfrac{10^{13}}{10^{4}}$
$= 0.315\times10^{9}$
$= 3.15\times10^{8}$
$= 315,000,000$
315,00,000 Americans

118. a. 1.89×10^{13}

b. 2.54×10^{11}

c. $\dfrac{1.89\times10^{13}}{2.54\times10^{11}} = \dfrac{1.89}{2.54}\times\dfrac{10^{13}}{10^{11}}$
$\approx 0.74\times10^{2}$
≈ 74
approximately 74 years

119. a. 1.09×10^{12}

b. 3.2×10^{7}

c. $\dfrac{1.09\times10^{12}}{3.2\times10^{7}} = \dfrac{1.09}{3.2}\times\dfrac{10^{12}}{10^{7}}$
$= 0.340625\times10^{5}$
$= 34,062.5$
34,062.5 years

120. – 128. Answers will vary.

129. does not make sense; Explanations will vary.
Sample explanation: $36(x^3)^9 = 36x^{27}$ not $36x^{12}$.

130. makes sense

131. does not make sense; Explanations will vary.
Sample explanation: 4.6×10^{12} represents over 4 trillion. The entire world population is measured in billions (10^9).

132. makes sense

133. false; Changes to make the statement true will vary.
A sample change is: $4^{-2} > 4^{-3}$.

134. true

135. false; Changes to make the statement true will vary.
A sample change is: $(-2)^4 \neq 2^{-4}$ because $16 \neq \dfrac{1}{16}$.

136. false; Changes to make the statement true will vary.
A sample change is: $5^2 \cdot 5^{-2} = 2^5 \cdot 2^{-5}$.

137. false; Changes to make the statement true will vary.
A sample change is: $534.7 \neq 5347$.

138. false; Changes to make the statement true will vary.
A sample change is:
$\dfrac{8\times10^{30}}{2\times10^{-5}} = 4\times10^{30-(-5)} = 4\times10^{35}$.

139. false; Changes to make the statement true will vary.
A sample change is:
$(7\times10^{5}) + (2\times10^{-3}) = 700,000.002$.

140. true

141. The doctor has gathered:
$2^{-1} + 2^{-2} = \dfrac{1}{2} + \dfrac{1}{2^2} = \dfrac{2}{4} + \dfrac{1}{4} = \dfrac{3}{4}$
So, $1 - \dfrac{3}{4} = \dfrac{1}{4}$ is remaining.

142. $b^A = MN, b^C = M, b^D = N$
$b^A = b^C b^D$
$A = C + D$

143. $\dfrac{70 \text{ bts}}{\cancel{\text{min}}} \cdot \dfrac{60 \cancel{\text{min}}}{\cancel{\text{hr}}} \cdot \dfrac{24 \cancel{\text{hrs}}}{\cancel{\text{day}}} \cdot \dfrac{365 \cancel{\text{days}}}{\cancel{\text{yr}}} \cdot 80 \cancel{\text{yrs}}$
$= 70 \cdot 60 \cdot 24 \cdot 365 \cdot 80$ beats
$= 2943360000$ beats
$= 2.94336\times10^{9}$ beats
$\approx 2.94\times10^{9}$ beats
The heartbeats approximately 2.94×10^{9} times over a lifetime of 80 years.

144. Answers will vary.

145. a. $\sqrt{16}\cdot\sqrt{4}=4\cdot 2=8$

b. $\sqrt{16\cdot 4}=\sqrt{64}=8$

c. $\sqrt{16}\cdot\sqrt{4}=\sqrt{16\cdot 4}$

146. a. $\sqrt{300}\approx 17.32$

b. $10\sqrt{3}\approx 17.32$

c. $\sqrt{300}=10\sqrt{3}$

147. a. $21x+10x=31x$

b. $21\sqrt{2}+10\sqrt{2}=31\sqrt{2}$

Section P.3

Check Point Exercises

1. a. $\sqrt{81}=9$

b. $-\sqrt{9}=-3$

c. $\sqrt{\frac{1}{25}}=\frac{1}{5}$

d. $\sqrt{36+64}=\sqrt{100}=10$

e. $\sqrt{36}+\sqrt{64}=6+8=14$

2. a. $\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

b.
$$\begin{aligned}\sqrt{5x}\cdot\sqrt{10x}&=\sqrt{5x\cdot 10x}\\&=\sqrt{50x^2}\\&=\sqrt{25\cdot 2x^2}\\&=\sqrt{25x^2}\cdot\sqrt{2}\\&=5x\sqrt{2}\end{aligned}$$

3. a. $\sqrt{\frac{25}{16}}=\frac{\sqrt{25}}{\sqrt{16}}=\frac{5}{4}$

b.
$$\begin{aligned}\frac{\sqrt{150x^3}}{\sqrt{2x}}&=\sqrt{\frac{150x^3}{2x}}\\&=\sqrt{75x^2}\\&=\sqrt{25x^2}\cdot\sqrt{3}\\&=5x\sqrt{3}\end{aligned}$$

4. a.
$$\begin{aligned}8\sqrt{13}+9\sqrt{13}&=(8+9)\sqrt{3}\\&=17\sqrt{13}\end{aligned}$$

b.
$$\begin{aligned}&\sqrt{17x}-20\sqrt{17x}\\&=1\sqrt{17x}-20\sqrt{17x}\\&=(1-20)\sqrt{17x}\\&=-19\sqrt{17x}\end{aligned}$$

5. a.
$$\begin{aligned}&5\sqrt{27}+\sqrt{12}\\&=5\sqrt{9\cdot 3}+\sqrt{4\cdot 3}\\&=5\cdot 3\sqrt{3}+2\sqrt{3}\\&=15\sqrt{3}+2\sqrt{3}\\&=(15+2)\sqrt{3}\\&=17\sqrt{3}\end{aligned}$$

b.
$$\begin{aligned}&6\sqrt{18x}-4\sqrt{8x}\\&=6\sqrt{9\cdot 2x}-4\sqrt{4\cdot 2x}\\&=6\cdot 3\sqrt{2x}-4\cdot 2\sqrt{2x}\\&=18\sqrt{2x}-8\sqrt{2x}\\&=(18-8)\sqrt{2x}\\&=10\sqrt{2x}\end{aligned}$$

6. a. If we multiply numerator and denominator by $\sqrt{3}$, the denominator becomes $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3$. Therefore, multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{9}}=\frac{5\sqrt{3}}{3}$$

b. The *smallest* number that will produce a perfect square in the denominator of $\frac{6}{\sqrt{12}}$ is $\sqrt{3}$ because $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$. So multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{6}{\sqrt{12}}=\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

7. Multiply by $\frac{4-\sqrt{5}}{4-\sqrt{5}}$.

$$\frac{8}{4+\sqrt{5}} = \frac{8}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$= \frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}$$
$$= \frac{8(4-\sqrt{5})}{16-5}$$
$$= \frac{8(4-\sqrt{5})}{11} \text{ or } \frac{32-8\sqrt{5}}{11}$$

8. **a.** $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$

b. $\sqrt[5]{8} \cdot \sqrt[5]{8} = \sqrt[5]{64} = \sqrt[5]{32} \cdot \sqrt[5]{2} = 2\sqrt[5]{2}$

c. $\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$

9. $3\sqrt[3]{81} - 4\sqrt[3]{3}$
$= 3\sqrt[3]{27 \cdot 3} - 4\sqrt[3]{3}$
$= 3 \cdot 3\sqrt[3]{3} - 4\sqrt[3]{3}$
$= 9\sqrt[3]{3} - 4\sqrt[3]{3}$
$= (9-4)\sqrt[3]{3}$
$= 5\sqrt[3]{3}$

10. **a.** $25^{\frac{1}{2}} = \sqrt{25} = 5$

b. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

c. $-81^{\frac{1}{4}} = -\sqrt[4]{81} = -3$

d. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

e. $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

11. **a.** $27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = (3)^4 = 81$

b. $4^{\frac{3}{2}} = \left(\sqrt[2]{4}\right)^3 = (2)^3 = 8$

c. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{\left(\sqrt[5]{32}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$

12. **a.** $\left(2x^{4/3}\right)\left(5x^{8/3}\right)$
$= 2 \cdot 5x^{4/3} \cdot x^{8/3}$
$= 10x^{(4/3)+(8/3)}$
$= 10x^{12/3}$
$= 10x^4$

b. $\frac{20x^4}{5x^{3/2}} = \left(\frac{20}{5}\right)\left(\frac{x^4}{x^{3/2}}\right)$
$= 4x^{4-(3/2)}$
$= 4x^{(8/2)-(3/2)}$
$= 4x^{5/2}$

13. $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

Concept and Vocabulary Check P.3

1. principal

2. 8^2

3. $|a|$

4. $\sqrt{a} \cdot \sqrt{b}$

5. $\frac{\sqrt{a}}{\sqrt{b}}$

6. $18\sqrt{3}$

7. 5; $6\sqrt{3}$

8. $7 - \sqrt{3}$

9. $\sqrt{10} + \sqrt{2}$

10. index; radicand

11. $(-2)^5$

12. a; $|a|$

13. $\sqrt[n]{a}$

14. 2; 8

Exercise Set P.3

1. $\sqrt{36} = \sqrt{6^2} = 6$

2. $\sqrt{25} = \sqrt{5^2} = 5$

3. $-\sqrt{36} = -\sqrt{6^2} = -6$

4. $-\sqrt{25} = -\sqrt{5^2} = -5$

5. $\sqrt{-36}$, The square root of a negative number is not real.

6. $\sqrt{-25}$, The square root of a negative number is not real.

7. $\sqrt{25-16} = \sqrt{9} = 3$

8. $\sqrt{144+25} = \sqrt{169} = 13$

9. $\sqrt{25} - \sqrt{16} = 5 - 4 = 1$

10. $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$

11. $\sqrt{(-13)^2} = \sqrt{169} = 13$

12. $\sqrt{(-17)^2} = \sqrt{289} = 17$

13. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

14. $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

15. $\sqrt{45x^2} = \sqrt{9x^2 \cdot 5}$
$= \sqrt{9x^2}\sqrt{5}$
$= \sqrt{9}\sqrt{x^2}\sqrt{5}$
$= 3|x|\sqrt{5}$

16. $\sqrt{125x^2} = \sqrt{25x^2 \cdot 5}$
$= \sqrt{25x^2}\sqrt{5}$
$= \sqrt{25}\sqrt{x^2}\sqrt{5}$
$= 5|x|\sqrt{5}$

17. $\sqrt{2x} \cdot \sqrt{6x} = \sqrt{2x \cdot 6x}$
$= \sqrt{12x^2}$
$= \sqrt{4x^2} \cdot \sqrt{3}$
$= 2x\sqrt{3}$

18. $\sqrt{10x} \cdot \sqrt{8x} = \sqrt{10x \cdot 8x}$
$= \sqrt{80x^2}$
$= \sqrt{16x^2} \cdot \sqrt{5}$
$= 4x\sqrt{5}$

19. $\sqrt{x^3} = \sqrt{x^2} \cdot \sqrt{x} = x\sqrt{x}$

20. $\sqrt{y^3} = \sqrt{y^2} \cdot \sqrt{y} = y\sqrt{y}$

21. $\sqrt{2x^2} \cdot \sqrt{6x} = \sqrt{2x^2 \cdot 6x}$
$= \sqrt{12x^3}$
$= \sqrt{4x^2} \cdot \sqrt{3x}$
$= 2x\sqrt{3x}$

22. $\sqrt{6x} \cdot \sqrt{3x^2} = \sqrt{6x \cdot 3x^2}$
$= \sqrt{18x^3}$
$= \sqrt{9x^2} \cdot \sqrt{2x}$
$= 3x\sqrt{2x}$

23. $\sqrt{\frac{1}{81}} = \frac{\sqrt{1}}{\sqrt{81}} = \frac{1}{9}$

24. $\sqrt{\frac{1}{49}} = \frac{\sqrt{1}}{\sqrt{49}} = \frac{1}{7}$

25. $\sqrt{\frac{49}{16}} = \frac{\sqrt{49}}{\sqrt{16}} = \frac{7}{4}$

26. $\sqrt{\frac{121}{9}} = \frac{\sqrt{121}}{\sqrt{9}} = \frac{11}{3}$

27. $\frac{\sqrt{48x^3}}{\sqrt{3x}} = \sqrt{\frac{48x^3}{3x}} = \sqrt{16x^2} = 4x$

28. $\frac{\sqrt{72x^3}}{\sqrt{8x}} = \sqrt{\frac{72x^3}{8x}} = \sqrt{9x^2} = 3x$

29. $\dfrac{\sqrt{150x^4}}{\sqrt{3x}} = \sqrt{\dfrac{150x^4}{3x}}$
$= \sqrt{50x^3}$
$= \sqrt{25x^2} \cdot \sqrt{2x}$
$= 5x\sqrt{2x}$

30. $\dfrac{\sqrt{24x^4}}{\sqrt{3x}} = \sqrt{\dfrac{24x^4}{3x}}$
$= \sqrt{8x^3}$
$= \sqrt{4x^2} \cdot \sqrt{2x}$
$= 2x\sqrt{2x}$

31. $\dfrac{\sqrt{200x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{200x^3}{10x^{-1}}}$
$= \sqrt{20x^{3-(-1)}}$
$= \sqrt{20x^4}$
$= \sqrt{4 \cdot 5x^4}$
$= 2x^2\sqrt{5}$

32. $\dfrac{\sqrt{500x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{500x^3}{10x^{-1}}} = \sqrt{50x^{3-(-1)}}$
$= \sqrt{50x^4} = \sqrt{25 \cdot 2x^4} = 5x^2\sqrt{2}$

33. $7\sqrt{3} + 6\sqrt{3} = (7+6)\sqrt{3} = 13\sqrt{3}$

34. $8\sqrt{5} + 11\sqrt{5} = (8+11)\sqrt{5} = 19\sqrt{5}$

35. $6\sqrt{17x} - 8\sqrt{17x} = (6-8)\sqrt{17x} = -2\sqrt{17x}$

36. $4\sqrt{13x} - 6\sqrt{13x} = (4-6)\sqrt{13x} = -2\sqrt{13x}$

37. $\sqrt{8} + 3\sqrt{2} = \sqrt{4 \cdot 2} + 3\sqrt{2}$
$= 2\sqrt{2} + 3\sqrt{2}$
$= (2+3)\sqrt{2}$
$= 5\sqrt{2}$

38. $\sqrt{20} + 6\sqrt{5} = \sqrt{4 \cdot 5} + 6\sqrt{5}$
$= 2\sqrt{5} + 6\sqrt{5}$
$= (2+6)\sqrt{5}$
$= 8\sqrt{5}$

39. $\sqrt{50x} - \sqrt{8x} = \sqrt{25 \cdot 2x} - \sqrt{4 \cdot 2x}$
$= 5\sqrt{2x} - 2\sqrt{2x}$
$= (5-2)\sqrt{2x}$
$= 3\sqrt{2x}$

40. $\sqrt{63x} - \sqrt{28x} = \sqrt{9 \cdot 7x} - \sqrt{4 \cdot 7x}$
$= 3\sqrt{7x} - 2\sqrt{7x}$
$= (3-2)\sqrt{7x}$
$= \sqrt{7x}$

41. $3\sqrt{18} + 5\sqrt{50} = 3\sqrt{9 \cdot 2} + 5\sqrt{25 \cdot 2}$
$= 3 \cdot 3\sqrt{2} + 5 \cdot 5\sqrt{2}$
$= 9\sqrt{2} + 25\sqrt{2}$
$= (9+25)\sqrt{2}$
$= 34\sqrt{2}$

42. $4\sqrt{12} - 2\sqrt{75} = 4\sqrt{4 \cdot 3} - 2\sqrt{25 \cdot 3}$
$= 4 \cdot 2\sqrt{3} - 2 \cdot 5\sqrt{3}$
$= 8\sqrt{3} - 10\sqrt{3}$
$= (8-10)\sqrt{3}$
$= -2\sqrt{3}$

43. $3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75}$
$= 3\sqrt{4 \cdot 2} - \sqrt{16 \cdot 2} + 3\sqrt{36 \cdot 2} - \sqrt{25 \cdot 3}$
$= 3 \cdot 2\sqrt{2} - 4\sqrt{2} + 3 \cdot 6\sqrt{2} - 5\sqrt{3}$
$= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3}$
$= 20\sqrt{2} - 5\sqrt{3}$

44. $3\sqrt{54} - 2\sqrt{24} - \sqrt{96} + 4\sqrt{63}$
$= 3\sqrt{9 \cdot 6} - 2\sqrt{4 \cdot 6} - \sqrt{16 \cdot 6} + 4\sqrt{9 \cdot 7}$
$= 3 \cdot 3\sqrt{6} - 2 \cdot 2\sqrt{6} - 4\sqrt{6} + 4 \cdot 3\sqrt{7}$
$= 9\sqrt{6} - 4\sqrt{6} - 4\sqrt{6} + 12\sqrt{7}$
$= \sqrt{6} + 12\sqrt{7}$

45. $\dfrac{1}{\sqrt{7}} = \dfrac{1}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{7}}{7}$

46. $\dfrac{2}{\sqrt{10}} = \dfrac{2}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{2\sqrt{10}}{10} = \dfrac{\sqrt{10}}{5}$

47. $\dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{2}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$

48. $\dfrac{\sqrt{7}}{\sqrt{3}} = \dfrac{\sqrt{7}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{21}}{3}$

49. $\dfrac{13}{3+\sqrt{11}} = \dfrac{13}{3+\sqrt{11}} \cdot \dfrac{3-\sqrt{11}}{3-\sqrt{11}}$

$$= \frac{13(3-\sqrt{11})}{3^2-(\sqrt{11})^2}$$

$$= \frac{13(3-\sqrt{11})}{9-11}$$

$$= \frac{13(3-\sqrt{11})}{-2}$$

50. $\dfrac{3}{3+\sqrt{7}} = \dfrac{3}{3+\sqrt{7}} \cdot \dfrac{3-\sqrt{7}}{3-\sqrt{7}}$

$$= \frac{3(3-\sqrt{7})}{3^2-(\sqrt{7})^2}$$

$$= \frac{3(3-\sqrt{7})}{9-7}$$

$$= \frac{3(3-\sqrt{7})}{2}$$

51. $\dfrac{7}{\sqrt{5}-2} = \dfrac{7}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$$= \frac{7(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}$$

$$= \frac{7(\sqrt{5}+2)}{5-4}$$

$$= 7(\sqrt{5}+2)$$

52. $\dfrac{5}{\sqrt{3}-1} = \dfrac{5}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \frac{5(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}$$

$$= \frac{5(\sqrt{3}+1)}{3-1}$$

$$= \frac{5(\sqrt{3}+1)}{2}$$

53. $\dfrac{6}{\sqrt{5}+\sqrt{3}} = \dfrac{6}{\sqrt{5}+\sqrt{3}} \cdot \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{6(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{6(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \frac{6(\sqrt{5}-\sqrt{3})}{2}$$

$$= 3(\sqrt{5}-\sqrt{3})$$

54. $\dfrac{11}{\sqrt{7}-\sqrt{3}} = \dfrac{11}{\sqrt{7}-\sqrt{3}} \cdot \dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$$= \frac{11(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$$

$$= \frac{11(\sqrt{7}+\sqrt{3})}{7-3}$$

$$= \frac{11(\sqrt{7}+\sqrt{3})}{4}$$

55. $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

56. $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

57. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

58. $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

59. $\sqrt[4]{-16}$ is not a real number.

60. $\sqrt[4]{-81}$ is not a real number.

61. $\sqrt[4]{(-3)^4} = |-3| = 3$

62. $\sqrt[4]{(-2)^4} = |-2| = 2$

63. $\sqrt[5]{(-3)^5} = -3$

64. $\sqrt[5]{(-2)^5} = -2$

65. $\sqrt[5]{-\dfrac{1}{32}} = \sqrt[5]{-\dfrac{1}{2^5}} = -\dfrac{1}{2}$

66. $\sqrt[6]{\dfrac{1}{64}} = \dfrac{\sqrt[6]{1}}{\sqrt[6]{2^6}} = \dfrac{1}{2}$

67. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\,\sqrt[3]{4} = 2 \cdot \sqrt[3]{4}$

68. $\sqrt[3]{150}$ cannot be simplified further.

69. $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = x \cdot \sqrt[3]{x}$

70. $\sqrt[3]{x^5} = \sqrt[3]{x^3 x^2} = x\sqrt[3]{x^2}$

71. $\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$

72. $\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

73. $\dfrac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} = \sqrt[5]{\dfrac{64x^6}{2x}} = \sqrt[5]{32x^5} = 2x$

74. $\dfrac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} = \sqrt[4]{\dfrac{162x^5}{2x}} = \sqrt[4]{81x^4} = 3x$

75. $4\sqrt[5]{2} + 3\sqrt[5]{2} = 7\sqrt[5]{2}$

76. $6\sqrt[5]{3} + 2\sqrt[5]{3} = 8\sqrt[5]{3}$

77. $5\sqrt[3]{16} + \sqrt[3]{54} = 5\sqrt[3]{8 \cdot 2} + \sqrt[3]{27 \cdot 2}$
$= 5 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 10\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

78. $3\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$
$= 3 \cdot 2\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 6\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 9\sqrt[3]{3}$

79. $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
$= \sqrt[3]{27 \cdot 2xy^3} - y\sqrt[3]{64 \cdot 2x}$
$= 3y\sqrt[3]{2x} - 4y\sqrt[3]{2x}$
$= -y\sqrt[3]{2x}$

80. $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$
$= \sqrt[3]{8 \cdot 3xy^3} - y\sqrt[3]{27 \cdot 3x}$
$= 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x}$
$= -y\sqrt[3]{3x}$

81. $\sqrt{2} + \sqrt[3]{8} = \sqrt{2} + 2$

82. $\sqrt{3} + \sqrt[3]{15}$ will not simplify.

83. $36^{1/2} = \sqrt{36} = 6$

84. $121^{1/2} = \sqrt{121} = 11$

85. $8^{1/3} = \sqrt[3]{8} = 2$

86. $27^{1/3} = \sqrt[3]{27} = 3$

87. $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

88. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

89. $32^{-4/5} = \dfrac{1}{32^{4/5}} = \dfrac{1}{2^4} = \dfrac{1}{16}$

90. $16^{-5/2} = \dfrac{1}{16^{5/2}} = \dfrac{1}{(\sqrt{16})^5} = \dfrac{1}{4^5} = \dfrac{1}{1024}$

91. $\left(7x^{1/3}\right)\left(2x^{1/4}\right) = 7 \cdot 2x^{1/3} \cdot x^{1/4}$
$= 14 \cdot x^{1/3+1/4}$
$= 14x^{7/12}$

92. $(3x^{2/3})(4x^{3/4}) = 3 \cdot 4x^{2/3} \cdot x^{3/4}$
$= 12 \cdot x^{2/3+3/4}$
$= 12x^{17/12}$

93. $\dfrac{20x^{1/2}}{5x^{1/4}} = \left(\dfrac{20}{5}\right)\left(\dfrac{x^{1/2}}{x^{1/4}}\right)$
$= 4 \cdot x^{1/2-1/4}$
$= 4x^{1/4}$

94. $\dfrac{72x^{3/4}}{9x^{1/3}} = \left(\dfrac{72}{9}\right)\left(\dfrac{x^{3/4}}{x^{1/3}}\right) = 8 \cdot x^{3/4-1/3} = 8x^{5/12}$

95. $\left(x^{2/3}\right)^3 = x^{2/3\cdot 3} = x^2$

96. $(x^{4/5})^5 = x^{4/5\cdot 5} = x^4$

97. $(25x^4y^6)^{1/2} = 25^{1/2}x^{4\cdot 1/2}y^{6\cdot 1/2} = 5x^2|y|^3$

98. $(125x^9y^6)^{1/3} = 125^{1/3}x^{9/3}y^{6/3} = 5x^3y^2$

99. $\dfrac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}} = \dfrac{27y^{\frac{3}{4}}}{y^{\frac{1}{12}}} = 27y^{\frac{3}{4}-\frac{1}{12}}$

$= 27y^{\frac{8}{12}} = 27y^{\frac{2}{3}}$

100. $\dfrac{\left(2y^{1/5}\right)^4}{y^{3/10}} = \dfrac{2^4\left(y^{1/5}\right)^4}{y^{3/10}}$

$= \dfrac{16y^{4/5}}{y^{3/10}} = 16y^{4/5-3/10} = 16y^{1/2}$

101. $\sqrt[4]{5^2} = 5^{2/4} = 5^{1/2} = \sqrt{5}$

102. $\sqrt[4]{7^2} = 7^{2/4} = 7^{1/2} = \sqrt{7}$

103. $\sqrt[3]{x^6} = x^{6/3} = x^2$

104. $\sqrt[4]{x^{12}} = x^{12/4} = |x|^3$

105. $\sqrt[6]{x^4} = \sqrt[6/2]{x^{4/2}} = \sqrt[3]{x^2}$

106. $\sqrt[9]{x^6} = \sqrt[9/3]{x^{6/3}} = \sqrt[3]{x^2}$

107. $\sqrt[9]{x^6y^3} = x^{\frac{6}{9}}y^{\frac{3}{9}} = x^{\frac{2}{3}}y^{\frac{1}{3}} = \sqrt[3]{x^2y}$

108. $\sqrt[12]{x^4y^8} = |x|^{\frac{4}{12}}|y|^{\frac{8}{12}} = |x|^{\frac{1}{3}}|y|^{\frac{2}{3}} = \sqrt[3]{|x|\,y^2}$

109. $\sqrt[3]{\sqrt[4]{16}+\sqrt{625}} = \sqrt[3]{2+25} = \sqrt[3]{27} = 3$

110. $\sqrt[3]{\sqrt{\sqrt{169}+\sqrt{9}}+\sqrt{\sqrt[3]{1000}+\sqrt[3]{216}}}$

$= \sqrt[3]{\sqrt{13+3}+\sqrt{10+6}}$

$= \sqrt[3]{\sqrt{16}+\sqrt{16}}$

$= \sqrt[3]{4+4} = \sqrt[3]{8}$

$= 2$

111. $\left(49x^{-2}y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$= (49)^{-1/2}\left(x^{-2}\right)^{-1/2}\left(y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$= \dfrac{1}{49^{1/2}}x^{(-2)(-1/2)}y^{(4)(-1/2)}\left(xy^{1/2}\right)$

$= \dfrac{1}{7}x^1y^{-2}\cdot xy^{1/2} = \dfrac{1}{7}x^{1+1}y^{-2+(1/2)}$

$= \dfrac{1}{7}x^2y^{-3/2} = \dfrac{x^2}{7y^{3/2}}$

112. $\left(8x^{-6}y^3\right)^{1/3}\left(x^{5/6}y^{-1/3}\right)^6$

$= 8^{1/3}x^{(-6)(1/3)}y^{(3)(1/3)}x^{(5/6)(6)}y^{(-1/3)(6)}$

$= 2x^{-2}y^1x^5y^{-2} = 2x^{-2+5}y^{1+(-2)}$

$= 2x^3y^{-1} = \dfrac{2x^3}{y}$

113. $\left(\dfrac{x^{-5/4}y^{1/3}}{x^{-3/4}}\right)^{-6} = \left(x^{(-5/4)-(-3/4)}y^{1/3}\right)^{-6}$

$= \left(x^{-2/4}y^{1/3}\right)^{-6} = x^{(-2/4)(-6)}y^{(1/3)(-6)}$

$= x^3y^{-2} = \dfrac{x^3}{y^2}$

114. $\left(\dfrac{x^{1/2}y^{-7/4}}{y^{-5/4}}\right)^{-4} = \left(x^{1/2}y^{(-7/4)-(-5/4)}\right)^{-4}$

$= \left(x^{1/2}y^{-2/4}\right)^{-4} = x^{(1/2)(-4)}y^{(-2/4)(-4)}$

$= x^{-2}y^2 = \dfrac{y^2}{x^2}$

115. The message is "Paige Fox is bad at math."

116. a. For 2030: $E = 5.8\sqrt{x} + 56.4$

$= 5.8\sqrt{10} + 56.4$

For 2060: $E = 5.8\sqrt{x} + 56.4$

$= 5.8\sqrt{40} + 56.4$

$= 5.8 \cdot 2\sqrt{10} + 56.4$

$= 11.6\sqrt{10} + 56.4$

Difference:

$(11.6\sqrt{10} + 56.4) - (5.8\sqrt{10} + 56.4)$

$= 11.6\sqrt{10} + 56.4 - 5.8\sqrt{10} - 56.4$

$= 11.6\sqrt{10} - 5.8\sqrt{10} + 56.4 - 56.4$

$= 5.8\sqrt{10}$

The difference is $5.8\sqrt{10}$.

b. $5.8\sqrt{10} \approx 18.3$

This underestimates the difference projected by the graph of $98.2 - 74.1 = 24.1$ by 5.8. This represents a difference of 5.8 million people.

117. $\dfrac{2}{\sqrt{5}-1} \cdot \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{2(\sqrt{5}+1)}{5-1}$

$= \dfrac{2(\sqrt{5}+1)}{4}$

$= \dfrac{\sqrt{5}+1}{2}$

≈ 1.62

About 1.62 to 1.

118. $R_a = R_f\sqrt{1-\left(\dfrac{v}{c}\right)^2}$

$= R_f\sqrt{1-\left(\dfrac{0.9c}{c}\right)^2}$

$= R_f\sqrt{1-(0.9)^2}$

$= R_f\sqrt{0.19}$

$\approx 0.44R_f$

$R_a = 0.44R_f$

$44 = 0.44R_f$

$\dfrac{44}{0.44} = \dfrac{0.44R_f}{0.44}$

$100 = R_f$

If you are gone for 44 weeks, then 100 weeks will have passed for your friend.

119. Perimeter:

$P = 2l + 2w$

$= 2 \cdot \sqrt{125} + 2 \cdot 2\sqrt{20}$

$= 2 \cdot \sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5}$

$= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5}$

$= 10\sqrt{5} + 8\sqrt{5}$

$= 18\sqrt{5}$ feet

Area:

$A = lw$

$= \sqrt{125} \cdot 2\sqrt{20}$

$= 2\sqrt{125 \cdot 20}$

$= 2\sqrt{2500}$

$= 2 \cdot 50$

$= 100$ square feet

120. Perimeter:

$P = 2l + 2w$

$= 2 \cdot 4\sqrt{20} + 2 \cdot \sqrt{80}$

$= 8\sqrt{4 \cdot 5} + 2\sqrt{16 \cdot 5}$

$= 8 \cdot 2\sqrt{5} + 2 \cdot 4\sqrt{5}$

$= 16\sqrt{5} + 8\sqrt{5}$

$= 24\sqrt{5}$ feet

Area:

$A = lw$

$= 4\sqrt{20} \cdot \sqrt{80}$

$= 4\sqrt{20 \cdot 80}$

$= 4\sqrt{1600}$

$= 4 \cdot 40$

$= 160$ square feet

121. – 128. Answers will vary.

129. does not make sense; Explanations will vary. Sample explanation: The denominator is rationalized correctly.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: $2\sqrt{20} + 4\sqrt{75}$ simplifies to $4\sqrt{5} + 20\sqrt{3}$ and thus the radical terms are not common.

132. does not make sense; Explanations will vary. Sample explanation: Finding the nth root first often gives smaller numbers on the middle step.

133. false; Changes to make the statement true will vary. A sample change is: $7^{1/2} \cdot 7^{1/2} = 7^1 = 7$.

134. false; Changes to make the statement true will vary. A sample change is: $(8)^{-\frac{1}{3}} = \dfrac{1}{(8)^{\frac{1}{3}}} = \dfrac{1}{\sqrt[3]{8}} = \dfrac{1}{2}$.

135. false; Changes to make the statement true will vary. The cube root of –8 is the real number –2.

136. false; Changes to make the statement true will vary. A sample change is: $\dfrac{\sqrt{20}}{8} = \dfrac{\sqrt{5}}{4}$.

137. $\left(5+\sqrt{\boxed{3}}\right)\left(5-\sqrt{\boxed{3}}\right) = 22$

$$25 - \boxed{3} = 22$$

$$\boxed{3} = 3$$

138. $\sqrt{\boxed{25}x^{\boxed{14}}} = 5x^7$

139. $\sqrt{13+\sqrt{2}+\dfrac{7}{3+\sqrt{2}}}$

$$= \sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}}}$$

$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{9-2}}$$

$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{7}}$$

$$= \sqrt{13+\sqrt{2}+3-\sqrt{2}}$$

$$= \sqrt{16}$$

$$= 4$$

140. a. $3^{\frac{1}{2}} \boxed{>} 3^{\frac{1}{3}}$

Calculator Check: $1.7321 > 1.4422$

b. $\sqrt{7}+\sqrt{18} \boxed{>} \sqrt{7+18}$

Calculator Check: $6.8884 > 5$

141. a. $\dfrac{ab}{a^2+ab+b^2} + \left(\dfrac{ac-ad-bc+bd}{ac-ad+bc-bd} \div \dfrac{a^3-b^3}{a^3+b^3}\right) = \dfrac{ab}{a^2+ab+b^2} + \left(\dfrac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)} \cdot \dfrac{a^3+b^3}{a^3-b^3}\right)$

$$= \frac{ab}{a^2+ab+b^2} + \left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}} \cdot \frac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right) = \frac{ab}{a^2+ab+b^2} + \frac{a^2-ab+b^2}{a^2+ab+b^2}$$

$$= \frac{ab+a^2-ab+b^2}{a^2+ab+b^2} = \frac{a^2+b^2}{a^2+ab+b^2}$$

Her son is 8 years old.

b. Son's portion:

$$\frac{8^{-\frac{4}{3}}+2^{-2}}{16^{-\frac{3}{4}}+2^{-1}}=\frac{\frac{1}{\left(\sqrt[3]{8}\right)^4}+\frac{1}{2^2}}{\frac{1}{\left(\sqrt[4]{16}\right)^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{2^4}+\frac{1}{4}}{\frac{1}{2^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{16}+\frac{1}{4}}{\frac{1}{8}+\frac{1}{2}}$$

$$=\frac{\frac{5}{16}}{\frac{5}{8}}$$

$$=\frac{8}{16}$$

$$=\frac{1}{2}$$

Mom's portion:

$$\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$$

142. $(2x^3y^2)(5x^4y^7)=10x^7y^9$

143. $2x^4(8x^4+3x)=2x^4(8x^4)+2x^4(3x)$

$$=16x^8+6x^5$$

144. $2x(x^2+4x+5)+3(x^2+4x+5)$

$=2x^3+8x^2+10x+3x^2+12x+15$

$=2x^3+8x^2+3x^2+10x+12x+15$

$=2x^3+11x^2+22x+15$

Section P.4

Check Point Exercises

1. **a.** $(-17x^3+4x^2-11x-5)+(16x^3-3x^2+3x-15)$

$=(-17x^3+16x^3)+(4x^2-3x^2)+(-11x+3x)+(-5-15)$

$=-x^3+x^2-8x-20$

b. $(13x^2-9x^2-7x+1)-(-7x^3+2x^2-5x+9)$
$=(13x^3-9x^2-7x+1)+(7x^3-2x^2+5x-9)$
$=(13x^3+7x^3)+(-9x^2-2x^2)+(-7x+5x)+(1-9)$
$=20x^3-11x^2-2x-8$

2. $(5x-2)(3x^2-5x+4)$
$=5x(3x^2-5x+4)-2(3x^2-5x+4)$
$=5x\cdot 3x^2-5x\cdot 5x+5x\cdot 4-2\cdot 3x^2+2\cdot 5x-2\cdot 4$
$=15x^3-25x^2+20x-6x^2+10x-8$
$=15x^3-31x^2+30x-8$

3. $(7x-5)(4x-3)=7x\cdot 4x+7x(-3)+(-5)4x+(-5)(-3)$
$=28x^2-21x-20x+15$
$=28x^2-41x+15$

4. **a.** Use the special-product formula shown.
$(A+B)(A-B)=A^2-B^2$
$(7x+8)(7x-8)=(7x)^2-(8)^2$
$=49x^2-64$

b. Use the special-product formula shown.
$(A+B)(A-B)=A^2-B^2$
$(2y^3-5)(2y^3+5)=(2y^3+5)(2y^3-5)$
$=(2y^3)^2-(5)^2$
$=4y^6-25$

5. **a.** Use the special-product formula shown.
$(A+B)^2=A^2+2AB+B^2$
$(x+10)^2=x^2+2(x)(10)+10^2$
$=x^2+20x+100$

b. Use the special-product formula shown.
$(A+B)^2=A^2+2AB+B^2$
$(5x+4)^2=(5x)^2+2(5x)(4)+4^2$
$=25x^2+40x+16$

6. **a.** Use the special-product formula shown.
$(A-B)^2=A^2-2AB+B^2$
$(x-9)^2=x^2-2(x)(9)+9^2$
$=x^2-18x+81$

b. Use the special-product formula shown.

$$(A-B)^2 = A^2 - 2AB + B^2$$
$$(7x-3)^2 = (7x)^2 - 2(7x)(3) + 3^2$$
$$= 49x^2 - 42x + 9$$

7. $(x^3 - 4x^2y + 5xy^2 - y^3) - (x^3 - 6x^2y + y^3)$
$= (x^3 - 4x^2y + 5xy^2 - y^3) + (-x^3 + 6x^2y - y^3)$
$= (x^3 - x^3) + (-4x^2y + 6x^2y) + (5xy^2) + (-y^3 - y^3)$
$= 2x^2y + 5xy^2 - 2y^3$

8. **a.** $(7x-6y)(3x-y) = (7x)(3x) + (7x)(-y) + (-6y)(3x) + (-6y)(-y)$
$= 21x^2 - 7xy - 18xy + 6y^2$
$= 21x^2 - 25xy + 6y^2$

b. $(2x+4y)^2 = (2x)^2 + 2(2x)(4y) + (4y)^2$
$= 4x^2 + 16xy + 16y^2$

Concept and Vocabulary Check P.4

1. whole

2. standard

3. monomial

4. binomial

5. trinomial

6. n

7. like;

8. distributive; $4x^3 - 8x^2 + 6$; $7x^3$

9. $5x$; 3; like

10. $3x^2$; $5x$; $21x$; 35

11. $A^2 - B^2$; minus

12. $A^2 + 2AB + B^2$; squared; product of the terms; squared

13. $A^2 - 2AB + B^2$; minus; product of the terms; plus

14. $n + m$

Exercise Set P.4

1. yes; $2x+3x^2-5=3x^2+2x-5$

2. no; The term $3x^{-1}$ does not have a whole number exponent.

3. no; The form of a polynomial involves addition and subtraction, not division.

4. yes; $x^2-x^3+x^4-5=x^4-x^3+x^2-5$

5. $3x^2$ has degree 2
$-5x$ has degree 1
4 has degree 0
$3x^2-5x+4$ has degree 2.

6. $-4x^3$ has degree 3
$7x^2$ has degree 2
-11 has degree 0
$-4x^3+7x^2-11$ has degree 3.

7. x^2 has degree 2
$-4x^3$ has degree 3
$9x$ has degree 1
$-12x^4$ has degree 4
63 has degree 0
$x^2-4x^3+9x-12x^4+63$ has degree 4.

8. x^2 has degree 2
$-8x^3$ has degree 3
$15x^4$ has degree 4
91 has degree 0
$x^2-8x^3+15x^4+91$ has degree 4.

9. $(-6x^3+5x^2-8x+9)+(17x^3+2x^2-4x-13)=(-6x^3+17x^3)+(5x^2+2x^2)+(-8x-4x)+(9-13)$
$=11x^3+7x^2-12x-4$
The degree is 3.

10. $(-7x^3+6x^2-11x+13)+(19x^3-11x^2+7x-17)=(-7x^3+19x^3)+(6x^2-11x^2)+(-11x+7x)+(13-17)$
$=12x^3-5x^2-4x-4$
The degree is 3.

11. $(17x^3-5x^2+4x-3)-(5x^3-9x^2-8x+11)=(17x^3-5x^2+4x-3)+(-5x^3+9x^2+8x-11)$
$=(17x^3-5x^3)+(-5x^2+9x^2)+(4x+8x)+(-3-11)$
$=12x^3+4x^2+12x-14$
The degree is 3.

12. $(18x^4-2x^3-7x+8)-(9x^4-6x^3-5x+7)=(18x^4-2x^3-7x+8)+(-9x^4+6x^3+5x-7)$
$=(18x^4-9x^4)+(-2x^3+6x^3)+(-7x+5x)+(8-7)$
$=9x^4+4x^3-2x+1$

The degree is 4.

13. $(5x^2-7x-8)+(2x^2-3x+7)-(x^2-4x-3)=(5x^2-7x-8)+(2x^2-3x+7)+(-x^2+4x+3)$
$=(5x^2+2x^2-x^2)+(-7x-3x+4x)+(-8+7+3)$
$=6x^2-6x+2$

The degree is 2.

14. $(8x^2+7x-5)-(3x^2-4x)-(-6x^3-5x^2+3)=(8x^2+7x-5)+(-3x^2+4x)+(6x^3+5x^2-3)$
$=6x^3+(8x^2-3x^2+5x^2)+(7x+4x)+(-5-3)$
$=6x^3+10x^2+11x-8$

The degree is 3.

15. $(x+1)(x^2-x+1)=x(x^2)-x\cdot x+x\cdot 1+1(x^2)-1\cdot x+1\cdot 1$
$=x^3-x^2+x+x^2-x+1$
$=x^3+1$

16. $(x+5)\left(x^2-5x+25\right)=x(x^2)-x(5x)+x(25)+5(x^2)-5(5x)+5(25)$
$=x^3-5x^2+25x+5x^2-25x+125)$
$=x^3+125$

17. $(2x-3)(x^2-3x+5)=(2x)(x^2)+(2x)(-3x)+(2x)(5)+(-3)(x^2)+(-3)(-3x)+(-3)(5)$
$=2x^3-6x^2+10x-3x^2+9x-15$
$=2x^3-9x^2+19x-15$

18. $(2x-1)(x^2-4x+3)=(2x)(x^2)+(2x)(-4x)+(2x)(3)+(-1)(x^2)+(-1)(-4x)+(-1)(3)$
$=2x^3-8x^2+6x-x^2+4x-3$
$=2x^3-9x^2+10x-3$

19. $(x+7)(x+3)=x^2+3x+7x+21=x^2+10x+21$

20. $(x+8)(x+5)=x^2+5x+8x+40=x^2+13x+40$

21. $(x-5)(x+3)=x^2+3x-5x-15=x^2-2x-15$

22. $(x-1)(x+2)=x^2+2x-x-2=x^2+x-2$

23. $(3x+5)(2x+1)=(3x)(2x)+3x(1)+5(2x)+5=6x^2+3x+10x+5=6x^2+13x+5$

24. $(7x+4)(3x+1)=(7x)(3x)+7x(1)+4(3x)+4(1)=21x^2+7x+12x+4=21x^2+19x+4$

25. $(2x-3)(5x+3)=(2x)(5x)+(2x)(3)+(-3)(5x)+(-3)(3)=10x^2+6x-15x-9=10x^2-9x-9$

26. $(2x-5)(7x+2) = (2x)(7x)+(2x)(2)+(-5)(7x)+(-5)(2) = 14x^2+4x-35x-10 = 14x^2-31x-10$

27. $(5x^2-4)(3x^2-7) = (5x^2)(3x^2)+(5x^2)(-7)+(-4)(3x^2)+(-4)(-7) = 15x^4-35x^2-12x^2+28 = 15x^4-47x^2+28$

28. $(7x^2-2)(3x^2-5) = (7x^2)(3x^2)+(7x^2)(-5)+(-2)(3x^2)+(-2)(-5) = 21x^4-35x^2-6x^2+10 = 21x^4-41x^2+10$

29. $\left(8x^3+3\right)\left(x^2-5\right) = \left(8x^3\right)\left(x^2\right)+\left(8x^3\right)(-5)+(3)\left(x^2\right)+(3)(-5) = 8x^5-40x^3+3x^2-15$

30. $\left(7x^3+5\right)\left(x^2-2\right) = \left(7x^3\right)\left(x^2\right)+\left(7x^3\right)(-2)+(5)\left(x^2\right)+(5)(-2) = 7x^5-14x^3+5x^2-10$

31. $(x+3)(x-3) = x^2-3^2 = x^2-9$

32. $(x+5)(x-5) = x^2-5^2 = x^2-25$

33. $(3x+2)(3x-2) = (3x)^2-2^2 = 9x^2-4$

34. $(2x+5)(2x-5) = (2x)^2-5^2 = 4x^2-25$

35. $(5-7x)(5+7x) = 5^2-(7x)^2 = 25-49x^2$

36. $(4-3x)(4+3x) = 4^2-(3x)^2 = 16-9x^2$

37. $(4x^2+5x)(4x^2-5x) = (4x^2)^2-(5x)^2 = 16x^4-25x^2$

38. $(3x^2+4x)(3x^2-4x) = (3x^2)^2-(4x)^2 = 9x^4-16x^2$

39. $\left(1-y^5\right)\left(1+y^5\right) = (1)^2-\left(y^5\right)^2 = 1-y^{10}$

40. $\left(2-y^5\right)\left(2+y^5\right) = (2)^2-\left(y^5\right)^2 = 4-y^{10}$

41. $(x+2)^2 = x^2+2\cdot x\cdot 2+2^2 = x^2+4x+4$

42. $(x+5)^2 = x^2+2\cdot x\cdot 5+5^2 = x^2+10x+25$

43. $(2x+3)^2 = (2x)^2+2(2x)(3)+3^2 = 4x^2+12x+9$

44. $(3x+2)^2 = (3x)^2+2(3x)(2)+2^2 = 9x^2+12x+4$

45. $(x-3)^2 = x^2-2\cdot x\cdot 3+3^2 = x^2-6x+9$

46. $(x-4)^2 = x^2-2\cdot x\cdot 4+4^2 = x^2-8x+16$

47. $(4x^2-1)^2 = (4x^2)^2-2(4x^2)(1)+1^2 = 16x^4-8x^2+1$

48. $(5x^2-3)^2 = (5x^2)^2-2(5x^2)(3)+3^2 = 25x^4-30x^2+9$

49. $(7-2x)^2 = 7^2 - 2(7)(2x) + (2x)^2 = 49 - 28x + 4x^2 = 4x^2 - 28x + 49$

50. $(9-5x)^2 = 9^2 - 2(9)(5x) + (5x)^2 = 81 - 90x + 25x^2$ or $25x^2 - 90x + 81$

51. $(x+1)^3 = x^3 + 3 \cdot x^2 \cdot 1 + 3x \cdot 1^2 + 1^3 = x^3 + 3x^2 + 3x + 1$

52. $(x+2)^3 = x^3 + 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 + 2^3 = x^3 + 6x^2 + 12x + 8$

53. $(2x+3)^3 = (2x)^3 + 3 \cdot (2x)^2 \cdot 3 + 3(2x) \cdot 3^2 + 3^3 = 8x^3 + 36x^2 + 54x + 27$

54. $(3x+4)^3 = (3x)^3 + 3(3x)^2 \cdot 4 + 3(3x) \cdot 4^2 + 4^3 = 27x^3 + 108x^2 + 144x + 64$

55. $(x-3)^3 = x^3 - 3 \cdot x^3 \cdot 3 + 3 \cdot x \cdot 3^2 - 3^3 = x^3 - 9x^2 + 27x - 27$

56. $(x-1)^3 = x^3 - 3x^2 \cdot 1 + 3x \cdot 1^2 - 1^3 = x^3 - 3x^2 + 3x - 1$

57. $(3x-4)^3 = (3x)^3 - 3(3x)^2 \cdot 4 + 3(3x) \cdot 4^2 - 4^3 = 27x^3 - 108x^2 + 144x - 64$

58. $(2x-3)^3 = (2x)^3 - 3(2x)^2 \cdot 3 + 3(2x) \cdot 3^2 - 3^3 = 8x^3 - 36x^2 + 54x - 27$

59.
$$\begin{aligned}(5x^2y - 3xy) + (2x^2y - xy) &= (5x^2y + 2x^2y) + (-3xy - xy) \\ &= (5+2)x^2y + (-3-1)xy \\ &= 7x^2y - 4xy \text{ is of degree 3.}\end{aligned}$$

60.
$$\begin{aligned}(-2x^2y + xy) + (4x^2y + 7xy) &= (-2x^2y + 4x^2y) + (xy + 7xy) \\ &= (-2+4)x^2y + (1+7)xy \\ &= 2x^2y + 8xy \text{ is of degree 3.}\end{aligned}$$

61.
$$\begin{aligned}(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2) &= (4x^2y - 2x^2y) + (8xy + 5xy) + (11+2) \\ &= (4-2)x^2y + (8+5)xy + 13 \\ &= 2x^2y + 13xy + 13 \text{ is of degree 3.}\end{aligned}$$

62.
$$\begin{aligned}(7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy) &= (7x^4y^2 - 18x^4y^2) + (-5x^2y^2 - 6x^2y^2) + (3xy - xy) \\ &= (7-18)x^4y^2 + (-5-6)x^2y^2 + (3-1)xy \\ &= -11x^4y^2 - 11x^2y^2 + 2xy \text{ is of degree 6.}\end{aligned}$$

63.
$$\begin{aligned}(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2) &= (x^3 + 7xy - 5y^2) \\ &= (x^3 - 6x^3) + (7xy + xy) + (-5y^2 - 4y^2) \\ &= (1-6)x^3 + (7+1)xy + (-5-4)y^2 \\ &= -5x^3 + 8xy - 9y^2 \text{ is of degree 3.}\end{aligned}$$

64. $(x^4 - 7xy - 5y^3) - (6x^4 - 3xy + 4y^3) = (x^4 - 7xy - 5y^3) + (-6x^4 + 3xy - 4y^3)$
$= (x^4 - 6x^4) + (-7xy + 3xy) + (-5y^3 - 4y^3)$
$= (1-6)x^4 + (-7+3)xy + (-5-4)y^3$
$= -5x^4 - 4xy - 9y^3$ is of degree 4.

65. $(3x^4y^2 + 5x^3y - 3y) - (2x^4y^2 - 3x^3y - 4y + 6x) = (3x^4y^2 + 5x^3y - 3y) + (-2x^4y^2 + 3x^3y + 4y - 6x)$
$= (3x^4y^2 - 2x^4y^2) + (5x^3y + 3x^3y) + (-3y + 4y) - 6x$
$= (3-2)x^4y^2 + (5+3)x^3y + (-3+4)y - 6x$
$= x^4y^2 + 8x^3y + y - 6x$ is of degree 6.

66. $(5x^4y^2 + 6x^3y - 7y) - (3x^4y^2 - 5x^3y - 6y + 8x) = (5x^4y^2 + 6x^3y - 7y) + (-3x^4y^2 + 5x^3y + 6y - 8x)$
$= (5x^4y^2 - 3x^4y^2) + (6x^3y + 5x^3y) + (-7y + 6y) - 8x$
$= (5-3)x^4y^2 + (6+5)x^3y + (-7+6)y - 8x$
$= 2x^4y^2 + 11x^3y - y - 8x$ is of degree 6.

67. $(x + 5y)(7x + 3y) = x(7x) + x(3y) + (5y)(7x) + (5y)(3y)$
$= 7x^2 + 3xy + 35xy + 15y^2$
$= 7x^2 + 38xy + 15y^2$

68. $(x + 9y)(6x + 7y) = x(6x) + x(7y) + (9y)(6x) + (9y)(7y)$
$= 6x^2 + 7xy + 54xy + 63y^2$
$= 6x^2 + 61xy + 63y^2$

69. $(x - 3y)(2x + 7y) = x(2x) + x(7y) + (-3y)(2x) + (-3y)(7y)$
$= 2x^2 + 7xy - 6xy - 21y^2$
$= 2x^2 + xy - 21y^2$

70. $(3x - y)(2x + 5y) = (3x)(2x) + (3x)(5y) + (-y)(2x) + (-y)(5y)$
$= 6x^2 + 15xy - 2xy - 5y^2$
$= 6x^2 + 13xy - 5y^2$

71. $(3xy - 1)(5xy + 2) = (3xy)(5xy) + (3xy)(2) + (-1)(5xy) + (-1)(2)$
$= 15x^2y^2 + 6xy - 5xy - 2$
$= 15x^2y^2 + xy - 2$

72. $(7x^2y + 1)(2x^2y - 3) = (7x^2y)(2x^2y) + (7x^2y)(-3) + (1)2x^2y + (1)(-3)$
$= 14x^4y^2 - 21x^2y + 2x^2y - 3$
$= 14x^4y^2 - 19x^2y - 3$

73. $(7x + 5y)^2 = (7x)^2 + 2(7x)(5y) + (5y)^2 = 49x^2 + 70xy + 25y^2$

74. $(9x + 7y)^2 = (9x)^2 + 2(9x)(7y) + (7y)^2 = 81x^2 + 126xy + 49y^2$

75. $(x^2y^2-3)^2=(x^2y^2)^2-2(x^2y^2)(3)+3^2=x^4y^4-6x^2y^2+9$

76. $(x^2y^2-5)^2=(x^2y^2)^2-2(x^2y^2)(5)+5^2=x^4y^4-10x^2y^2+25$

77. $(x-y)(x^2+xy+y^2)=x(x^2)+x(xy)+x(y^2)+(-y)(x^2)+(-y)(xy)+(-y)(y^2)$

$=x^3+x^2y+xy^2-x^2y-xy^2-y^3$

$=x^3-y^3$

78. $(x+y)(x^2-xy+y^2)=x(x^2)+x(-xy)+x(y^2)+y(x^2)+y(-xy)+y(y^2)$

$=x^3-x^2y+xy^2+x^2y-xy^2+y^3$

$=x^3+y^3$

79. $(3x+5y)(3x-5y)=(3x)^2-(5y)^2=9x^2-25y^2$

80. $(7x+3y)(7x-3y)=(7x)^2-(3y)^2=49x^2-9y^2$

81. $\left(7xy^2-10y\right)\left(7xy^2+10y\right)=\left(7xy^2\right)^2-\left(10y\right)^2=49x^2y^4-100y^2$

82. $\left(3xy^2-4y\right)\left(3xy^2+4y\right)=\left(3xy^2\right)^2-\left(4y\right)^2=9x^2y^4-16y^2$

83. $(3x+4y)^2-(3x-4y)^2=\left[(3x)^2+2(3x)(4y)+(4y)^2\right]-\left[(3x)^2-2(3x)(4y)+(4y)^2\right]$

$=\left(9x^2+24xy+16y^2\right)-\left(9x^2-24xy+16y^2\right)$

$=9x^2+24xy+16y^2-9x^2+24xy-16y^2$

$=48xy$

84. $(5x+2y)^2-(5x-2y)^2=\left[(5x)^2+2(5x)(2y)+(2y)^2\right]-\left[(5x)^2-2(5x)(2y)+(2y)^2\right]$

$=\left(25x^2+20xy+4y^2\right)-\left(25x^2-20xy+4y^2\right)$

$=25x^2+20xy+4y^2-25x^2+20xy-4y^2$

$=40xy$

85. $(5x-7)(3x-2)-(4x-5)(6x-1)$

$=\left[15x^2-10x-21x+14\right]-\left[24x^2-4x-30x+5\right]$

$=\left(15x^2-31x+14\right)-\left(24x^2-34x+5\right)$

$=15x^2-31x+14-24x^2+34x-5$

$=-9x^2+3x+9$

86. $(3x+5)(2x-9)-(7x-2)(x-1)$

$=\left(6x^2-27x+10x-45\right)-\left(7x^2-7x-2x+2\right)$

$=\left(6x^2-17x-45\right)-\left(7x^2-9x+2\right)$

$=6x^2-17x-45-7x^2+9x-2$

$=-x^2-8x-47$

87. $(2x+5)(2x-5)\left(4x^2+25\right)$

$=\left[(2x)^2-5^2\right]\left(4x^2+25\right)$

$=\left(4x^2-25\right)\left(4x^2+25\right)$

$=\left(4x^2\right)^2-(25)^2$

$=16x^4-625$

88. $(3x+4)(3x-4)\left(9x^2+16\right)$

$=\left[(3x)^2-4^2\right]\left(9x^2+16\right)$

$=\left(9x^2-16\right)\left(9x^2+16\right)$

$=\left(9x^2\right)^2-(16)^2$

$=81x^4-256$

89. $\dfrac{(2x-7)^5}{(2x-7)^3}=(2x-7)^{5-3}$

$=(2x-7)^2$

$=(2x)^2-2(2x)(7)+(7)^2$

$=4x^2-28x+49$

90. $\dfrac{(5x-3)^6}{(5x-3)^4}=(5x-3)^{6-4}$

$=(5x-3)^2$

$=(5x)^2-2(5x)(3)+(3)^2$

$=25x^2-30x+9$

91. **a.** $S=0.2x^3-1.5x^2+3.4x+25+\left(0.1x^3-1.3x^2+3.3x+5\right)$

$S=0.2x^3-1.5x^2+3.4x+25+0.1x^3-1.3x^2+3.3x+5$

$S=0.3x^3-2.8x^2+6.7x+30$

b. $S = 0.3x^3 - 2.8x^2 + 6.7x + 30$

$S = 0.3(5)^3 - 2.8(5)^2 + 6.7(5) + 30$

$S = 31$

The model gives a score of 31 for the group in the 45-54 age range which is the same as the score displayed by the bar graph.

92. **a.** $S = -0.02x^3 + 0.4x^2 + 1.2x + 22 + \left(-0.01x^3 - 0.2x^2 + 1.1x + 2\right)$

$S = -0.02x^3 + 0.4x^2 + 1.2x + 22 - 0.01x^3 - 0.2x^2 + 1.1x + 2$

$S = -0.03x^3 + 0.2x^2 + 2.3x + 24$

b. $S = -0.03x^3 + 0.2x^2 + 2.3x + 24$

$S = -0.03(5)^3 + 0.2(5)^2 + 2.3(5) + 24$

$S = 36.75$

The model gives a score of 36.75 for the group of slightly conservative political identification group. This underestimates the score shown on the bar graph by 0.25.

93. $x(8-2x)(10-2x) = x\left(80 - 36x + 4x^2\right)$

$= 80x - 36x^2 + 4x^3$

$= 4x^3 - 36x^2 + 80x$

94. $x(8-2x)(5-2x) = x\left(40 - 26x + 4x^2\right)$

$= 40x - 26x^2 + 4x^3$

$= 4x^3 - 26x^2 + 40x$

95. $(x+9)(x+3) - (x+5)(x+1)$

$= x^2 + 12x + 27 - \left(x^2 + 6x + 5\right)$

$= x^2 + 12x + 27 - x^2 - 6x - 5$

$= 6x + 22$

96. $(x+4)(x+3) - (x+2)(x+1)$

$= x^2 + 7x + 12 - \left(x^2 + 3x + 2\right)$

$= x^2 + 7x + 12 - x^2 - 3x - 2$

$= 4x + 10$

97. – 102. Answers will vary.

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: FOIL is used to multiply two binomials.

105. makes sense

106. makes sense, although answers may vary

107. false; Changes to make the statement true will vary. A sample change is: $(3x^3 + 2)(3x^3 - 2) = 9x^6 - 4$

108. false; Changes to make the statement true will vary. A sample change is: $(x-5)^2 = x^2 - 10x + 25$

109. false; Changes to make the statement true will vary. A sample change is: $(x+1)^2 = x^2 + 2x + 1$

110. true

111. $[(7x+5)+4y][(7x+5)-4y] = (7x+5)^2 - 4y^2$
$= (7x)^2 + 2(7x)(5) + 5^2 - 16y^2$
$= 49x^2 + 70x + 25 - 16y^2$

112. $[(3x+y)+1]^2$
$= (3x+y)^2 + 2(3x+y)(1) + 1^2$
$= (3x)^2 + 2(3x)y + y^2 + 6x + 2y + 1$
$= 9x^2 + 6xy + y^2 + 6x + 2y + 1$

113. $(x^n+2)(x^n-2) - (x^n-3)^2$
$(x^n+2)(x^n-2) - (x^n-3)^2$
$= (x^{2n} - 4) - (x^{2n} - 6x^n + 9)$
$= x^{2n} - 4 - x^{2n} + 6x^n - 9$
$= 6x^n - 13$

114. $(x+3)(x-1) + ((x+3)-x)(x-(x-1))$
$= (x+3)(x-1) + 3(x-x+1)$
$= x^2 - x + 3x - 3 + 3$
$= x^2 + 2x$

115. $(x+3)(x+\boxed{4}) = x^2 + 7x + 12$

116. $(x-\boxed{2})(x-12) = x^2 - 14x + 24$

117. $(4x+1)(2x-\boxed{3}) = 8x^2 - 10x - 3$

Mid-Chapter P Check Point

1. $(3x+5)(4x-7) = (3x)(4x) + (3x)(-7) + (5)(4x) + (5)(-7)$
$= 12x^2 - 21x + 20x - 35$
$= 12x^2 - x - 35$

2. $(3x+5) - (4x-7) = 3x + 5 - 4x + 7$
$= 3x - 4x + 5 + 7$
$= -x + 12$

3. $\sqrt{6} + 9\sqrt{6} = 10\sqrt{6}$

4. $3\sqrt{12} - \sqrt{27} = 3 \cdot 2\sqrt{3} - 3\sqrt{3} = 6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$

5. $7x+3\left[9-(2x-6)\right]=7x+3\left[9-2x+6\right]=7x+3\left[15-2x\right]=7x+45-6x=x+45$

6. $(8x-3)^2=(8x)^2-2(8x)(3)+(3)^2=64x^2-48x+9$

7. $\left(x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^6=x^{\frac{1}{3}\cdot 6}y^{-\frac{1}{2}\cdot 6}=x^2y^{-3}=\dfrac{x^2}{y^3}$

8. $\left(\dfrac{2}{7}\right)^0-32^{-\frac{2}{5}}=1-\dfrac{1}{\left(\sqrt[5]{32}\right)^2}=1-\dfrac{1}{(2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

9. $(2x-5)-(x^2-3x+1)=2x-5-x^2+3x-1=-x^2+5x-6$

10.
$$\begin{aligned}(2x-5)(x^2-3x+1)&=2x(x^2-3x+1)-5(x^2-3x+1)\\&=2x(x^2-3x+1)-5(x^2-3x+1)\\&=2x^3-6x^2+2x-5x^2+15x-5\\&=2x^3-6x^2-5x^2+2x+15x-5\\&=2x^3-11x^2+17x-5\end{aligned}$$

11. $x^3+x^3-x^3\cdot x^3=2x^3-x^6=-x^6+2x^3$

12.
$$\begin{aligned}(9a-10b)(2a+b)&=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\&=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\&=18a^2+9ab-20ab-10b^2\\&=18a^2-11ab-10b^2\end{aligned}$$

13. $\{a,c,d,e\}\cup\{c,d,f,h\}=\{a,c,d,e,f,h\}$

14. $\{a,c,d,e\}\cap\{c,d,f,h\}=\{c,d\}$

15.
$$\begin{aligned}\left(3x^2y^3-xy+4y^2\right)-\left(-2x^2y^3-3xy+5y^2\right)&=3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\&=3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\&=3x^2y^3+2x^2y^3-xy+3xy+4y^2-5y^2\\&=5x^2y^3+2xy-y^2\end{aligned}$$

16. $\dfrac{24x^2y^{13}}{-2x^5y^{-2}}=-12x^{2-5}y^{13-(-2)}=-12x^{-3}y^{15}=-\dfrac{12y^{15}}{x^3}$

17. $\left(\dfrac{1}{3}x^{-5}y^4\right)\left(18x^{-2}y^{-1}\right)=6x^{-5-2}y^{4-1}=\dfrac{6y^3}{x^7}$

18. $\sqrt[12]{x^4}=x^{\frac{4}{12}}=\left|x^{\frac{1}{3}}\right|=\left|\sqrt[3]{x}\right|$

19. $\dfrac{24\times10^3}{2\times10^6}=\dfrac{24}{2}\cdot\dfrac{10^3}{10^6}=12\times10^{-3}=\left(1.2\times10^1\right)\times10^{-3}=1.2\times\left(10^1\times10^{-3}\right)=1.2\times10^{-2}$

20. $\dfrac{\sqrt[3]{32}}{\sqrt[3]{2}}=\sqrt[3]{\dfrac{32}{2}}=\sqrt[3]{16}=\sqrt[3]{2^4}=2\sqrt[3]{2}$

21. $(x^3+2)(x^3-2)=x^6-4$

22. $(x^2+2)^2=(x^2)^2+2(x^2)(2)+(2)^2=x^4+4x^2+4$

23. $\sqrt{50}\cdot\sqrt{6}=5\sqrt{2}\cdot\sqrt{6}=5\sqrt{2\cdot6}=5\sqrt{12}=5\cdot2\sqrt{3}=10\sqrt{3}$

24. $\dfrac{11}{7-\sqrt{3}}=\dfrac{11}{7-\sqrt{3}}\cdot\dfrac{7+\sqrt{3}}{7+\sqrt{3}}=\dfrac{77+11\sqrt{3}}{49-3}=\dfrac{77+11\sqrt{3}}{46}$

25. $\dfrac{11}{\sqrt{3}}=\dfrac{11}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{11\sqrt{3}}{3}$

26. $\left\{-11,\ -\dfrac{3}{7},\ 0,\ 0.45,\ \sqrt{25}\right\}$

27. Since $2-\sqrt{13}<0$ then $\left|2-\sqrt{13}\right|=\sqrt{13}-2$

28. Since $x<0$ then $|x|=-x$. Thus $x^2|x|=-x^2x=-x^3$

29. $4.6\cdot3.0\times10^8=4.6\times10^8=13.8\times10^8=1.38\times10^9$
The U.S. produces 1.38×10^9 pounds of garbage per day.

30. $\dfrac{3\times10^{10}}{7.5\times10^9}=\dfrac{3}{7.5}\cdot\dfrac{10^{10}}{10^9}=0.4\times10=4$
A human brain has 4 times as many neurons as a gorilla brain.

31. **a.** Model 1:
$D=1188x+16,218$
$D=1188(1)+16,218$
$D=17,406$
Model 2:
$D=46x^2+541x+17,650$
$D=46(1)^2+541(1)+17,650$
$D=18,237$

Model 1 best describes the data in 2001.

b. $D=46x^2+541x+17,650$
$D=46(13)^2+541(13)+17,650$
$D=32,457$
Model 2 underestimates the average student-loan debt in 2013 by \$593.

Section P.5

Check Point Exercises

1. **a.** $10x^3 - 4x^2$
$= 2x^2(5x) - 2x^2(2)$
$= 2x^2(5x - 2)$

b. $2x(x-7) + 3(x-7)$
$= (x-7)(2x+3)$

2. $x^3 + 5x^2 - 2x - 10$
$= (x^3 + 5x^2) - (2x + 10)$
$= x^2(x+5) - 2(x+5)$
$= (x+5)(x^2 - 2)$

3. Find two numbers whose product is 40 and whose sum is 13. The required integers are 8 and 5. Thus, $x^2 + 13x + 40 = (x+8)(x+5)$ or $(x+5)(x+8)$.

4. Find two numbers whose product is –14 and whose sum is –5. The required integers are –7 and 2. Thus, $x^2 - 5x - 14 = (x-7)(x+2)$ or $(x+2)(x-7)$.

5. Find two First terms whose product is $6x^2$.
$6x^2 + 19x - 7 = (6x \quad)(x \quad)$
$6x^2 + 19x - 7 = (3x \quad)(2x \quad)$

Find two Last terms whose product is –7.
The possible factors are 1(–7) and –1(7).

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $19x$.

Possible Factors of $6x^2 + 19x - 7$	Sum of Outside and Inside Products (Should Equal $19x$)
$(6x+1)(x-7)$	$-42x + x = -41x$
$(6x-7)(x+1)$	$6x - 7x = -x$
$(6x-1)(x+7)$	$42x - x = 41x$
$(6x+7)(x-1)$	$-6x + 7x = x$
$(3x+1)(2x-7)$	$-21x + 2x = -19x$
$(3x-7)(2x+1)$	$3x - 14x = -11x$
$(3x-1)(2x+7)$	$21x - 2x = 19x$
$(3x+7)(2x-1)$	$-3x + 14x = 11x$

Thus, $6x^2 + 19x - 7 = (3x-1)(2x+7)$ or $(2x+7)(3x-1)$.

6. Find two First terms whose product is $3x^2$.
$3x^2 - 13xy + 4y^2 = (3x \quad)(x \quad)$

Find two Last terms whose product is $4y^2$.
The possible factors are $(2y)(2y)$, $(-2y)(-2y)$, $(4y)(y)$, and $(-4y)(-y)$.

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $-13xy$.

$3x^2 - 13xy + y^2 = (3x - y)(x - 4y)$ or $(x - 4y)(3x - y)$.

7. Express each term as the square of some monomial. Then use the formula for factoring $A^2 - B^2$.

a. $x^2 - 81 = x^2 - 9^2 = (x + 9)(x - 9)$

b. $36x^2 - 25 = (6x)^2 - 5^2 = (6x + 5)(6x - 5)$

8. Express $81x^4 - 16$ as the difference of two squares and use the formula for factoring $A^2 - B^2$.
$81x^4 - 16 = (9x^2)^2 - 4^2 = (9x^2 + 4)(9x^2 - 4)$

The factor $9x^2 - 4$ is the difference of two squares and can be factored. Express $9x^2 - 4$ as the difference of two squares and again use the formula for factoring $A^2 - B^2$.
$(9x^2 + 4)(9x^2 - 4) = (9x^2 + 4)\left[(3x)^2 - 2^2\right] = (9x^2 + 4)(3x + 2)(3x - 2)$

Thus, factored completely,
$81x^4 - 16 = (9x^2 + 4)(3x + 2)(3x - 2)$.

9. **a.** $x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2 = (x + 7)^2$

b. Since $16x^2 = (4x)^2$ and $49 = 7^2$, check to see if the middle term can be expressed as twice the product of $4x$ and 7. Since $2 \cdot 4x \cdot 7 = 56x$, $16x^2 - 56x + 49$ is a perfect square trinomial. Thus, $16x^2 - 56x + 49 = (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2$
$= (4x - 7)^2$

10. **a.** $x^3 + 1 = x^3 + 1^3$
$= (x + 1)(x^2 - x \cdot 1 + 1^2)$
$= (x + 1)(x^2 - x + 1)$

b. $125x^3 - 8 = (5x)^3 - 2^3$
$= (5x - 2)\left[(5x)^2 + (5x)(2) + 2^2\right]$
$= (5x - 2)(25x^2 + 10x + 4)$

11. Factor out the greatest common factor.
$3x^3 - 30x^2 + 75x = 3x\left(x^2 - 10x + 25\right)$
Factor the perfect square trinomial.
$3x\left(x^2 - 10x + 25\right) = 3x(x - 5)^2$

12. Reorder to write as a difference of squares.

$$\begin{aligned}
&x^2 - 36a^2 + 20x + 100 \\
&= x^2 + 20x + 100 - 36a^2 \\
&= \left(x^2 + 20x + 100\right) - 36a^2 \\
&= \left(x+10\right)^2 - 36a^2 \\
&= \left(x+10+6a\right)\left(x+10-6a\right)
\end{aligned}$$

13. $x\left(x-1\right)^{-\frac{1}{2}} + \left(x-1\right)^{\frac{1}{2}}$

$$\begin{aligned}
&= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right] \\
&= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)\right] \\
&= \left(x-1\right)^{-\frac{1}{2}}\left(2x-1\right) \\
&= \frac{2x-1}{\left(x-1\right)^{\frac{1}{2}}}
\end{aligned}$$

Concept and Vocabulary Check P.5

1. d

2. g

3. b

4. c

5. c

6. a

7. f

8. $(x+1)^{\frac{1}{2}}$

Exercise Set P.5

1. $18x + 27 = 9 \cdot 2x + 9 \cdot 3 = 9(2x+3)$

2. $16x - 24 = 8(2x) + 8(-3) = 8(2x - 3)$

3. $3x^2 + 6x = 3x \cdot x + 3x \cdot 2 = 3x(x+2)$

4. $4x^2 - 8x = 4x(x) + 4x(-2) = 4x(x - 2)$

5.
$$\begin{aligned}
&9x^4 - 18x^3 + 27x^2 \\
&= 9x^2(x^2) + 9x^2(-2x) + 9x^2(3) \\
&= 9x^2(x^2 - 2x + 3)
\end{aligned}$$

6.
$$\begin{aligned}
&6x^4 - 18x^3 + 12x^2 \\
&= 6x^2(x^2) + 6x^2(-3x) + 6x^2(2) \\
&= 6x^2\left(x^2 - 3x + 2\right)
\end{aligned}$$

7. $x(x + 5) + 3(x + 5) = (x + 5)(x + 3)$

8. $x(2x + 1) + 4(2x + 1) = (2x + 1)(x + 4)$

9. $x^2(x-3) + 12(x-3) = (x-3)(x^2+12)$

10. $x^2\left(2x+5\right) + 17\left(2x+5\right) = \left(2x+5\right)\left(x^2+17\right)$

11.
$$\begin{aligned}
x^3 - 2x^2 + 5x - 10 &= x^2(x-2) + 5(x-2) \\
&= (x^2+5)(x-2)
\end{aligned}$$

12.
$$\begin{aligned}
x^3 - 3x^2 + 4x - 12 &= x^2\left(x-3\right) + 4\left(x-3\right) \\
&= \left(x-3\right)\left(x^2+4\right)
\end{aligned}$$

13.
$$\begin{aligned}
x^3 - x^2 + 2x - 2 &= x^2(x-1) + 2(x-1) \\
&= (x-1)(x^2+2)
\end{aligned}$$

14.
$$\begin{aligned}
x^3 + 6x^2 - 2x - 12 &= x^2\left(x+6\right) - 2\left(x+6\right) \\
&= \left(x+6\right)\left(x^2-2\right)
\end{aligned}$$

15.
$$\begin{aligned}
3x^3 - 2x^2 - 6x + 4 &= x^2(3x-2) - 2(3x-2) \\
&= (3x-2)(x^2-2)
\end{aligned}$$

16.
$$\begin{aligned}
x^3 - x^2 - 5x + 5 &= x^2\left(x-1\right) - 5\left(x-1\right) \\
&= \left(x-1\right)\left(x^2-5\right)
\end{aligned}$$

17. $x^2 + 5x + 6 = (x+2)(x+3)$

18. $x^2 + 8x + 15 = (x+3)(x+5)$

19. $x^2 - 2x - 15 = (x - 5)(x + 3)$

20. $x^2 - 4x - 5 = (x-5)(x+1)$

21. $x^2 - 8x + 15 = (x - 5)(x - 3)$

22. $x^2 - 14x + 45 = (x-5)(x-9)$

23. $3x^2 - x - 2 = (3x + 2)(x - 1)$

24. $2x^2+5x-3=(2x-1)(x+3)$

25. $3x^2-25x-28=(3x-28)(x+1)$

26. $3x^2-2x-5=(3x-5)(x+1)$

27. $6x^2-11x+4=(2x-1)(3x-4)$

28. $6x^2-17x+12=(2x-3)(3x-4)$

29. $4x^2+16x+15=(2x+3)(2x+5)$

30. $8x^2+33x+4=(8x+1)(x+4)$

31. $9x^2-9x+2=(3x-1)(3x-2)$

32. $9x^2+5x-4=(9x-4)(x+1)$

33. $20x^2+27x-8=(5x+8)(4x-1)$

34. $15x^2-19x+6=(3x-2)(5x-3)$

35. $2x^2+3xy+y^2=(2x+y)(x+y)$

36. $3x^2+4xy+y^2=(3x+y)(x+y)$

37. $6x^2-5xy-6y^2=(3x+2y)(2x-3y)$

38. $6x^2-7xy-5y^2=(3x-5y)(2x+y)$

39. $x^2-100=x^2-10^2=(x+10)(x-10)$

40. $x^2-144=x^2-12^2=(x+12)(x-12)$

41. $36x^2-49=(6x)^2-7^2=(6x+7)(6x-7)$

42. $64x^2-81=(8x)^2-9^2=(8x+9)(8x-9)$

43. $9x^2-25y^2=(3x)^2-(5y)^2$
$=(3x+5y)(3x-5y)$

44. $36x^2-49y^2=(6x)^2-(7y)^2$
$=(6x+7y)(6x-7y)$

45. $x^4-16=(x^2)^2-4^2$
$=(x^2+4)(x^2-4)$
$=(x^2+4)(x+2)(x-2)$

46. $x^4-1=(x^2)^2-1^2=(x^2+1)(x^2-1)$
$=(x^2+1)(x+1)(x-1)$

47. $16x^4-81=(4x^2)^2-9^2$
$=(4x^2+9)(4x^2-9)$
$=(4x^2+9)[(2x)^2-3^2]$
$=(4x^2+9)(2x+3)(2x-3)$

48. $81x^4-1=(9x^2)^2-1^2$
$=(9x^2+1)(9x^2-1)$
$=(9x^2+1)[(3x)^2-1^2]$
$=(9x^2+1)(3x+1)(3x-1)$

49. $x^2+2x+1=x^2+2\cdot x\cdot 1+1^2=(x+1)^2$

50. $x^2+4x+4=x^2+2\cdot x\cdot 2+2^2=(x+2)^2$

51. $x^2-14x+49=x^2-2\cdot x\cdot 7+7^2$
$=(x-7)^2$

52. $x^2-10x+25=x^2-2\cdot x\cdot 5+5^2=(x-5)^2$

53. $4x^2+4x+1=(2x)^2+2\cdot 2x\cdot 1+1^2$
$=(2x+1)^2$

54. $25x^2+10x+1=(5x)^2+2\cdot 5x\cdot 1+1^2=(5x+1)^2$

55. $9x^2-6x+1=(3x)^2-2\cdot 3x\cdot 1+1^2$
$=(3x-1)^2$

56. $64x^2-16x+1=(8x)^2-2\cdot 8x\cdot 1+1^2=(8x-1)^2$

57. $x^3+27=x^3+3^3$
$=(x+3)(x^2-x\cdot 3+3^2)$
$=(x+3)(x^2-3x+9)$

58. $x^3+64=x^3+4^3$
$=(x+4)(x^2-x\cdot 4+4^2)$
$=(x+4)(x^2-4x+16)$

59. $x^3-64=x^3-4^3$
$=(x-4)(x^2+x\cdot 4+4^2)$
$=(x-4)(x^2+4x+16)$

60. $x^3 - 27 = x^3 - 3^3$
$= (x-3)(x^2 + x\cdot 3 + 3^2)$
$= (x-3)(x^2 + 3x + 9)$

61. $8x^3 - 1 = (2x)^3 - 1^3$
$= (2x-1)[(2x)^2 + (2x)(1) + 1^2]$
$= (2x-1)(4x^2 + 2x + 1)$

62. $27x^3 - 1 = (3x)^3 - 1^3$
$= (3x-1)[(3x)^2 + (3x)(1) + 1^2]$
$= (3x-1)(9x^2 + 3x + 1)$

63. $64x^3 + 27 = (4x)^3 + 3^3$
$= (4x+3)[(4x)^2 - (4x)(3) + 3^2]$
$= (4x+3)(16x^2 - 12x + 9)$

64. $8x^3 + 125 = (2x)^3 + 5^3$
$= (2x+5)[(2x)^2 - (2x)(5) + 5^2]$
$= (2x+5)(4x^2 - 10x + 25)$

65. $3x^3 - 3x = 3x(x^2 - 1) = 3x(x+1)(x-1)$

66. $5x^3 - 45x = 5x(x^2 - 9) = 5x(x+3)(x-3)$

67. $4x^2 - 4x - 24 = 4(x^2 - x - 6)$
$= 4(x+2)(x-3)$

68. $6x^2 - 18x - 60 = 6(x^2 - 3x - 10)$
$= 6(x+2)(x-5)$

69. $2x^4 - 162 = 2(x^4 - 81)$
$= 2[(x^2)^2 - 9^2]$
$= 2(x^2 + 9)(x^2 - 9)$
$= 2(x^2 + 9)(x^2 - 3^2)$
$= 2(x^2 + 9)(x+3)(x-3)$

70. $7x^4 - 7 = 7(x^4 - 1)$
$= 7[(x^2)^2 - 1^2]$
$= 7(x^2 + 1)(x^2 - 1)$
$= 7(x^2 + 1)(x+1)(x-1)$

71. $x^3 + 2x^2 - 9x - 18 = (x^3 + 2x^2) - (9x + 18)$
$= x^2(x+2) - 9(x+2)$
$= (x^2 - 9)(x+2)$
$= (x^2 - 3^2)(x+2)$
$= (x-3)(x+3)(x+2)$

72. $x^3 + 3x^2 - 25x - 75 = (x^3 + 3x^2) - (25x + 75)$
$= x^2(x+3) - 25(x+3)$
$= (x^2 - 25)(x+3)$
$= (x^2 - 5^2)(x+3)$
$= (x-5)(x+5)(x+3)$

73. $2x^2 - 2x - 112 = 2(x^2 - x - 56) = 2(x-8)(x+7)$

74. $6x^2 - 6x - 12 = 6(x^2 - x - 2)$
$= 6(x-2)(x+1)$

75. $x^3 - 4x = x(x^2 - 4)$
$= x(x^2 - 2^2)$
$= x(x-2)(x+2)$

76. $9x^3 - 9x = 9x(x^2 - 1) = 9x(x-1)(x+1)$

77. $x^2 + 64$ is prime.

78. $x^2 + 36$ is prime.

79. $x^3+2x^2-4x-8=(x^3+2x^2)+(-4x-8)$
$=x^2(x+2)-4(x+2)=(x^2-4)(x+2)=(x^2-2^2)(x+2)=(x-2)(x+2)(x+2)=(x-2)(x+2)^2$

80. x^3+2x^2-x-2
$=(x^3+2x^2)+(-x-2)=x^2(x+2)-1(x+2)=(x^2-1)(x+2)=(x^2-1^2)(x+2)=(x-1)(x+1)(x+2)$

81. y^5-81y
$=y(y^4-81)=y[(y^2)^2-9^2]=y(y^2+9)(y^2-9)=y(y^2+9)(y^2-3^2)=y(y^2+9)(y+3)(y-3)$

82. y^5-16y
$=y(y^4-16)=y[(y^2)^2-4^2]=y(y^2+4)(y^2-4)=y(y^2+4)(y^2-2^2)=y(y^2+4)(y+2)(y-2)$

83. $20y^4-45y^2=5y^2(4y^2-9)=5y^2[(2y)^2-3^2]=5y^2(2y+3)(2y-3)$

84. $48y^4-3y^2=3y^2(16y^2-1)=3y^2[(4y)^2-1^2]=3y^2(4y+1)(4y-1)$

85. $x^2-12x+36-49y^2=\left(x^2-12x+36\right)-49y^2=\left(x-6\right)^2-49y^2=(x-6+7y)\left(x-6-7y\right)$

86. $x^2-10x+25-36y^2=(x^2-10x+25)-36y^2=(x-5)^2-36y^2=\left(x-5+6y\right)\left(x-5-6y\right)$

87. $9b^2x-16y-16x+9b^2y$
$=\left(9b^2x+9b^2y\right)+\left(-16x-16y\right)=9b^2\left(x+y\right)-16\left(x+y\right)=\left(x+y\right)\left(9b^2-16\right)=\left(x+y\right)\left(3b+4\right)\left(3b-4\right)$

88. $16a^2x-25y-25x+16a^2y$
$=\left(16a^2x+16a^2y\right)+\left(-25y-25x\right)=16a^2\left(x+y\right)-25\left(x+y\right)=\left(x+y\right)\left(16a^2-25\right)=\left(x+y\right)\left(4a+5\right)\left(4a-5\right)$

89. $x^2y-16y+32-2x^2$
$=\left(x^2y-16y\right)+\left(-2x^2+32\right)=y\left(x^2-16\right)-2\left(x^2-16\right)=\left(x^2-16\right)\left(y-2\right)=\left(x+4\right)\left(x-4\right)\left(y-2\right)$

90. $12x^2y-27y-4x^2+9$
$=\left(12x^2y-27y\right)+\left(-4x^2+9\right)=3y\left(4x^2-9\right)-1\left(4x^2-9\right)=\left(4x^2-9\right)\left(3y-1\right)=\left(2x+3\right)\left(2x-3\right)\left(3y-1\right)$

91. $2x^3-8a^2x+24x^2+72x$
$=2x\left(x^2-4a^2+12x+36\right)=2x\left[\left(x^2+12x+36\right)-4a^2\right]=2x\left[\left(x+6\right)^2-4a^2\right]=2x\left(x+6-2a\right)\left(x+6+2a\right)$

92. $2x^3-98a^2x+28x^2+98x$
$=2x\left(x^2-49a^2+14x+49\right)=2x\left[\left(x^2+14x+49\right)-49a^2\right]=2x\left[\left(x+7\right)^2-49a^2\right]=2x\left(x+7-7a\right)\left(x+7+7a\right)$

93. $x^{\frac{3}{2}}-x^{\frac{1}{2}}=x^{\frac{1}{2}}\left(x^{\frac{3}{2}-\frac{1}{2}}\right)-1=x^{\frac{1}{2}}\left(x-1\right)$

94. $x^{\frac{3}{4}} - x^{\frac{1}{4}} = x^{\frac{1}{4}}\left(x^{\frac{3}{4}-\frac{1}{4}} - 1\right) = x^{\frac{1}{4}}\left(x^{\frac{1}{2}} - 1\right)$

95. $4x^{-\frac{2}{3}} + 8x^{\frac{1}{3}} = 4x^{-\frac{2}{3}}\left(1 + 2x^{\frac{1}{3}-\left(-\frac{2}{3}\right)}\right) = 4x^{-\frac{2}{3}}(1+2x) = \dfrac{4(1+2x)}{x^{\frac{2}{3}}}$

96. $12x^{-\frac{3}{4}} + 6x^{\frac{1}{4}} = 6x^{-\frac{3}{4}}\left(2 + x^{\frac{1}{4}-\left(-\frac{3}{4}\right)}\right) = 6x^{-\frac{3}{4}}(2+x) = \dfrac{6(x+2)}{x^{\frac{3}{4}}}$

97. $(x+3)^{\frac{1}{2}} - (x+3)^{\frac{3}{2}} = (x+3)^{\frac{1}{2}}\left[1 - (x+3)^{\frac{3}{2}-\frac{1}{2}}\right] = (x+3)^{\frac{1}{2}}\left[1-(x+3)\right] = (x+3)^{\frac{1}{2}}(-x-2) = -(x+3)^{\frac{1}{2}}(x+2)$

98. $\left(x^2+4\right)^{\frac{3}{2}} + \left(x^2+4\right)^{\frac{7}{2}} = \left(x^2+4\right)^{\frac{3}{2}}\left[1 + \left(x^2+4\right)^{\frac{7}{2}-\frac{3}{2}}\right] = \left(x^2+4\right)^{\frac{3}{2}}\left[1 + \left(x^2+4\right)^2\right] = \left(x^2+4\right)^{\frac{3}{2}}\left(x^4+8x^2+17\right)$

99. $(x+5)^{-\frac{1}{2}} - (x+5)^{-\frac{3}{2}} = (x+5)^{-\frac{3}{2}}\left[(x+5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)} - 1\right] = (x+5)^{-\frac{3}{2}}\left[(x+5)-1\right] = (x+5)^{-\frac{3}{2}}(x+4) = \dfrac{x+4}{(x+5)^{\frac{3}{2}}}$

100. $\left(x^2+3\right)^{-\frac{2}{3}} + \left(x^2+3\right)^{-\frac{5}{3}} = \left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)^{-\frac{2}{3}-\left(-\frac{5}{3}\right)} + 1\right] = \left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)+1\right] = \dfrac{x^2+4}{(x^2+3)^{5/3}}$

101. $(4x-1)^{\frac{1}{2}} - \dfrac{1}{3}(4x-1)^{\frac{3}{2}}$

$= (4x-1)^{\frac{1}{2}}\left[1 - \dfrac{1}{3}(4x-1)^{\frac{3}{2}-\frac{1}{2}}\right] = (4x-1)^{\frac{1}{2}}\left[1 - \dfrac{1}{3}(4x-1)\right] = (4x-1)^{\frac{1}{2}}\left[1 - \dfrac{4}{3}x + \dfrac{1}{3}\right]$

$= (4x-1)^{\frac{1}{2}}\left(\dfrac{4}{3} - \dfrac{4}{3}x\right) = (4x-1)^{\frac{1}{2}}\dfrac{4}{3}(1-x) = \dfrac{-4(4x-1)^{\frac{1}{2}}(x-1)}{3}$

102. $-8(4x+3)^{-2} + 10(5x+1)(4x+3)^{-1} = 2(4x+3)^{-2}\left[-4 + 5(5x+1)(4x+3)\right] = \dfrac{2(100x^2+95x+11)}{(4x+3)^2}$

103. $10x^2(x+1) - 7x(x+1) - 6(x+1) = (x+1)\left(10x^2 - 7x - 6\right) = (x+1)(5x-6)(2x+1)$

104. $12x^2(x-1) - 4x(x-1) - 5(x-1) = (x-1)\left(12x^2 - 4x - 5\right) = (x-1)(6x-5)(2x+1)$

105. $6x^4 + 35x^2 - 6 = \left(x^2+6\right)\left(6x^2-1\right)$

106. $7x^4 + 34x^2 - 5 = \left(7x^2-1\right)\left(x^2+5\right)$

107. $y^7 + y = y\left(y^6 + 1\right) = y\left[\left(y^2\right)^3 + 1^3\right] = y\left(y^2 + 1\right)\left(y^4 - y^2 + 1\right)$

108. $(y+1)^3 + 1 = (y+1)^3 + 1^3 = \left[(y+1)+1\right]\left[(y+1)^2 - (y+1) + 1\right] = (y+2)\left[\left(y^2 + 2y + 1\right) - y - 1 + 1\right]$
$= (y+2)\left(y^2 + 2y + 1 - y - 1 + 1\right) = (y+2)\left(y^2 + y + 1\right)$

109. $x^4 - 5x^2y^2 + 4y^4 = \left(x^2 - 4y^2\right)\left(x^2 - y^2\right) = (x+2y)(x-2y)(x+y)(x-y)$

110. $x^4 - 10x^2y^2 + 9y^4 = \left(x^2 - 9y^2\right)\left(x^2 - y^2\right) = (x+3y)(x-3y)(x+y)(x-y)$

111. $(x-y)^4 - 4(x-y)^2$
$= (x-y)^2\left((x-y)^2 - 4\right) = (x-y)^2\left((x-y)+2\right)\left((x-y)-2\right) = (x-y)^2(x-y+2)(x-y-2)$

112. $(x+y)^4 - 100(x+y)^2 = (x+y)^2\left((x+y)^2 - 100\right) = (x+y)^2(x+y-10)(x+y+10)$

113. $2x^2 - 7xy^2 + 3y^4 = \left(2x - y^2\right)\left(x - 3y^2\right)$

114. $3x^2 + 5xy^2 + 2y^4 = \left(3x + 2y^2\right)\left(x + y^2\right)$

115. a. $(x - 0.4x) - 0.4(x - 0.4x) = (x - 0.4x)(1 - 0.4) = (0.6x)(0.6) = 0.36x$

b. No, the computer is selling at 36% of its original price.

116. a. $(x - 0.3x) - 0.3(x - 0.3x) = (x - 0.3x)(1 - 0.3) = (0.7x)(0.7) = 0.49x$

b. No, the computer is selling at 49% of its original price.

117. a. $(3x)^2 - 4 \cdot 2^2 = 9x^2 - 16$

b. $9x^2 - 16 = (3x+4)(3x-4)$

118. a. $(7x)^2 - 4 \cdot 3^2 = 49x^2 - 36$

b. $49x^2 - 36 = (7x+6)(7x-6)$

119. a. $x(x+y) - y(x+y)$

b. $x(x+y) - y(x+y) = (x+y)(x-y)$

120. a. $x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

b. $x^2 + 2xy + y^2 = (x+y)^2$

121. $V_{\text{shaded}} = V_{\text{outside}} - V_{\text{inside}}$
$= a \cdot a \cdot 4a - b \cdot b \cdot 4a$
$= 4a^3 - 4ab^2$
$= 4a\left(a^2 - b^2\right)$
$= 4a(a+b)(a-b)$

122. $V_{shaded} = V_{outside} - V_{inside}$
$= a \cdot a \cdot 3a - b \cdot b \cdot 3a$
$= 3a^3 - 3ab^2$
$= 3a\left(a^2 - b^2\right)$
$= 3a(a+b)(a-b)$

123. – 129. Answers will vary.

130. makes sense

131. makes sense

132. does not make sense; Explanations will vary. Sample explanation: $4x^2 - 100 = 4(x^2 - 25) = 4(x+5)(x-5)$

133. makes sense

134. false; Changes to make the statement true will vary. A sample change is:
$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x+2)(x-2)$

135. true

136. false; Changes to make the statement true will vary. A sample change is: The binomial $x^2 + 36$ is prime.

137. false; Changes to make the statement true will vary. A sample change is: $x^3 - 64 = (x-4)(x+4x+16)$

138. $x^{2n} + 6x^n + 8 = \left(x^n + 4\right)\left(x^n + 2\right)$

139. $-x^2 - 4x + 5 = -1\left(x^2 + 4x - 5\right) = -1(x+5)(x-1) = -(x+5)(x-1)$

140. $x^4 - y^4 - 2x^3y + 2xy^3$
$= \left(x^4 - y^4\right) + \left(-2x^3y + 2xy^3\right)$
$= \left(x^2 - y^2\right)\left(x^2 + y^2\right) - 2xy\left(x^2 - y^2\right)$
$= \left(x^2 - y^2\right)\left(x^2 + y^2 - 2xy\right)$
$= (x-y)(x+y)\left(x^2 - 2xy + y^2\right)$
$= (x-y)(x+y)(x-y)^2$
$= (x-y)^3(x+y)$

141. $(x-5)^{-\frac{1}{2}}(x+5)^{-\frac{1}{2}}-(x+5)^{\frac{1}{2}}(x-5)^{-\frac{3}{2}}=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-(x+5)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right]$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)-(x+5)\right]$$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}(-10)=\frac{-10}{(x-5)^{3/2}(x+5)^{1/2}}$$

142. $x^2+bx+15$, b =16, – 16, 8 or –8

143. b = 0, 3, 4, or $-c(c+4)$, where $c > 0$ is an integer.

144. $\dfrac{x^2+6x+5}{x^2-25}=\dfrac{(x+5)(x+1)}{(x+5)(x-5)}=\dfrac{x+1}{x-5}$

145. $\dfrac{5}{4}\cdot\dfrac{8}{15}=\dfrac{5}{4}\cdot\dfrac{4\cdot2}{5\cdot3}$

$$=\frac{1}{1}\cdot\frac{2}{3}$$

$$=\frac{2}{3}$$

146. $\dfrac{1}{2}+\dfrac{2}{3}=\dfrac{3}{6}+\dfrac{4}{6}$

$$=\frac{7}{6}$$

Section P.6

Check Point Exercises

1. a. The denominator would equal zero if $x=-5$, so –5 must be excluded from the domain.

b. $x^2-36=(x+6)(x-6)$

The denominator would equal zero if $x=-6$ or x = 6, so –6 and 6 both must be excluded from the domain.

c. $x^2-5x-14=(x+2)(x-7)$

The denominator would equal zero if $x=-2$ or x = 7, so –2 and 7 both must be excluded from the domain.

2. a. $\dfrac{x^3+3x^2}{x+3}=\dfrac{x^2(x+3)}{x+3}$

$$=\frac{x^2(x+3)}{x+3}$$

$$=x^2,\ x\neq-3$$

Because the denominator is x + 3, $x\neq-3$

b. $\dfrac{x^2-1}{x^2+2x+1}=\dfrac{(x-1)(x+1)}{(x+1)(x+1)}=\dfrac{x-1}{x+1},x\neq-1$

Because the denominator is $(x+1)(x+1), x\neq-1$

3. $\dfrac{x+3}{x^2-4}\cdot\dfrac{x^2-x-6}{x^2+6x+9}$

$=\dfrac{x+3}{(x+2)(x-2)}\cdot\dfrac{(x-3)(x+2)}{(x+3)(x+3)}$

$=\dfrac{x+3}{(x+2)(x-2)}\cdot\dfrac{(x-3)(x+2)}{(x+3)(x+3)}$

$=\dfrac{x-3}{(x-2)(x+3)},\ x\neq -2,\ x\neq 2,\ x\neq -3$

Because the denominator has factors of $x+2,\ x-2,$ and $x+3,\ x\neq -2,\ x\neq 2,$ and $x\neq -3.$

4. $\dfrac{x^2-2x+1}{x^3+x}\div\dfrac{x^2+x-2}{3x^2+3}$

$=\dfrac{x^2-2x+1}{x^3+x}\cdot\dfrac{3x^2+3}{x^2+x-2}$

$=\dfrac{(x-1)(x-1)}{x(x^2+1)}\cdot\dfrac{3(x^2+1)}{(x+2)(x-1)}$

$=\dfrac{3(x-1)}{x(x+2)},\ x\neq 1,\ x\neq 0,\ x\neq -2$

5. $\dfrac{x}{x+1}-\dfrac{3x+2}{x+1}=\dfrac{x-3x-2}{x+1}$

$=\dfrac{-2x-2}{x+1}$

$=\dfrac{-2(x+1)}{x+1}$

$=-2, x\neq -1$

6. Factor each denominator completely.

$x+1=1(x+1)$

$x-1=1(x-1)$

List the factors of the first denominator.

$1,\ x+1$

Add any unlisted factors from the second denominator.

$1,\ x+1,\ x-1$

The least common denominator is the product of all factors in the final list.

$1(x+1)(x-1)$ or $(x+1)(x-1)$ is the least common denominator.

7. Factor each denominator completely.

$x^2-6x+9=(x-3)^2$

$x^2-9=(x+3)(x-3)$

List the factors of the first denominator.

$x-3,\ x-3$

Add any unlisted factors from the second denominator.

$x-3,\ x-3,\ x+3$

The least common denominator is the product of all factors in the final list.

$(x-3)(x-3)(x+3)$ or $(x-3)^2(x+3)$ is the least common denominator.

8. Find the least common denominator.

$x-3=1(x-3)$

$x+3=1(x+3)$

The least common denominator is $(x-3)(x+3)$. Write all rational expressions in terms of the least common denominator.

$\dfrac{x}{x-3}+\dfrac{x-1}{x+3}$

$=\dfrac{x(x+3)}{(x-3)(x+3)}+\dfrac{(x-1)(x-3)}{(x-3)(x+3)}$

Add numerators, putting this sum over the least common denominator.

$=\dfrac{x(x+3)+(x-1)(x-3)}{(x-3)(x+3)}$

$=\dfrac{x^2+3x+\left(x^2-4x+3\right)}{(x-3)(x+3)}$

$=\dfrac{x^2+3x+x^2-4x+3}{(x-3)(x+3)}$

$=\dfrac{2x^2-x+3}{(x-3)(x+3)},\ x\neq 3, x\neq -3$

9. Find the least common denominator.

$x^2-10x+25=(x-5)^2$

$2x-10=2(x-5)$

The least common denominator is $2(x-5)(x-5)$. Write all rational expressions in terms of the least common denominator.

$$\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}$$
$$=\frac{x}{(x-5)(x-5)}-\frac{x-4}{2(x-5)}$$
$$=\frac{2x}{2(x-5)(x-5)}-\frac{(x-4)(x-5)}{2(x-5)(x-5)}$$

Add numerators, putting this sum over the least common denominator.

$$=\frac{2x-(x-4)(x-5)}{2(x-5)(x-5)}$$
$$=\frac{2x-(x^2-5x-4x+20)}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)^2},\ x\neq 5$$

10.
$$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}=\frac{\frac{2}{2x}-\frac{3x}{2x}}{\frac{4}{4x}+\frac{3x}{4x}},\ x\neq 0$$
$$=\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}},\ x\neq\frac{-4}{3}$$
$$=\frac{2-3x}{2x}\div\frac{4+3x}{4x}$$
$$=\frac{2-3x}{2x}\cdot\frac{4x}{4+3x}$$
$$=\frac{2-3x}{4+3x}\cdot\frac{4}{2}$$
$$=\frac{2-3x}{4+3x}\cdot\frac{2}{1}$$
$$=\frac{2(2-3x)}{4+3x},\ x\neq 0,\ x\neq\frac{-4}{3}$$

11. Multiply each of the three terms, $\frac{1}{x+7}$, $\frac{1}{x}$, and 7 by the least common denominator of $x(x+7)$.

$$\frac{\frac{1}{x+7}-\frac{1}{x}}{7}=\frac{x(x+7)\left(\frac{1}{x+7}\right)-x(x+7)\left(\frac{1}{x}\right)}{7x(x+7)}$$
$$=\frac{x-(x+7)}{7x(x+7)}$$
$$=\frac{-7}{7x(x+7)}$$
$$=-\frac{1}{x(x+7)},\ x\neq 0,\ x\neq -7$$

Concept and Vocabulary Check P.6

1. polynomials

2. domain; 0

3. factoring; common factors

4. $\frac{x^2}{15}$

5. $\frac{3}{5}$

6. $\frac{x^2-x+4}{3}$

7. $x+3$ and $x-2$; $x+3$ and $x+1$; $(x+3)(x-2)(x+1)$

8. $3x+4$

9. complex; complex

10. x; $x+3$; -3; $\frac{1}{x(x+3)}$

Exercise Set P.6

1. $\frac{7}{x-3}, x\neq 3$

2. $\frac{13}{x+9}, x\neq -9$

3. $\dfrac{x+5}{x^2-25}=\dfrac{x+5}{(x+5)(x-5)}, x\neq 5,-5$

4. $\dfrac{x+7}{x^2-49}=\dfrac{x+7}{(x+7)(x-7)}, x\neq 7,-7$

5. $\dfrac{x-1}{x^2+11x+10}=\dfrac{x-1}{(x+1)(x+10)}, x\neq -1,-10$

6. $\dfrac{x-3}{x^2+4x-45}=\dfrac{x-3}{(x+9)(x-5)}, x\neq -9,5$

7. $\dfrac{3x-9}{x^2-6x+9}=\dfrac{3(x-3)}{(x-3)(x-3)}$

$=\dfrac{3}{x-3}, x\neq 3$

8. $\dfrac{4x-8}{x^2-4x+4}=\dfrac{4(x-2)}{(x-2)(x-2)}=\dfrac{4}{x-2}, \ x\neq 2$

9. $\dfrac{x^2-12x+36}{4x-24}=\dfrac{(x-6)(x-6)}{4(x-6)}=\dfrac{x-6}{4}.$

$x\neq 6$

10. $\dfrac{x^2-8x+16}{3x-12}=\dfrac{(x-4)(x-4)}{3(x-4)}=\dfrac{x-4}{3}, \ x\neq 4$

11. $\dfrac{y^2+7y-18}{y^2-3y+2}=\dfrac{(y+9)(y-2)}{(y-2)(y-1)}=\dfrac{y+9}{y-1},$

$y\neq 1, 2$

12. $\dfrac{y^2-4y-5}{y^2+5y+4}=\dfrac{(y-5)(y+1)}{(y+4)(y+1)}=\dfrac{y-5}{y+4}, y\neq -4,-1$

13. $\dfrac{x^2+12x+36}{x^2-36}=\dfrac{(x+6)^2}{(x+6)(x-6)}=\dfrac{x+6}{x-6},$

$x\neq 6, -6$

14. $\dfrac{x^2-14x+49}{x^2-49}=\dfrac{(x-7)^2}{(x-7)(x+7)}$

$=\dfrac{x-7}{x+7},$

$x\neq 7, -7$

15. $\dfrac{x-2}{3x+9}\cdot\dfrac{2x+6}{2x-4}=\dfrac{x-2}{3(x+3)}\cdot\dfrac{2(x+3)}{2(x-2)}$

$=\dfrac{2}{6}=\dfrac{1}{3}, x\neq 2,-3$

16. $\dfrac{6x+9}{3x-15}\cdot\dfrac{x-5}{4x+6}=\dfrac{3(2x+3)}{3(x-5)}\cdot\dfrac{x-5}{2(2x+3)}$

$=\dfrac{3}{6}$

$=\dfrac{1}{2},$

$x\neq 5,-\dfrac{3}{2}$

17. $\dfrac{x^2-9}{x^2}\cdot\dfrac{x^2-3x}{x^2+x-12}$

$=\dfrac{(x-3)(x+3)}{x^2}\cdot\dfrac{x(x-3)}{(x+4)(x-3)}$

$=\dfrac{(x-3)(x+3)}{x(x+4)}, x\neq 0,-4,3$

18. $\dfrac{x^2-4}{x^2-4x+4}\cdot\dfrac{2x-4}{x+2}=\dfrac{(x+2)(x-2)}{(x-2)^2}\cdot\dfrac{2(x-2)}{x+2}$

$=2,$

$x\neq 2, -2$

19. $\dfrac{x^2-5x+6}{x^2-2x-3}\cdot\dfrac{x^2-1}{x^2-4}$

$=\dfrac{(x-3)(x-2)}{(x-3)(x+1)}\cdot\dfrac{(x+1)(x-1)}{(x-2)(x+2)}$

$=\dfrac{x-1}{x+2}, \ x\neq -2,-1,2,3$

20. $\dfrac{x^2+5x+6}{x^2+x-6}\cdot\dfrac{x^2-9}{x^2-x-6}$

$=\dfrac{(x+3)(x+2)}{(x+3)(x-2)}\cdot\dfrac{(x-3)(x+3)}{(x-3)(x+2)}=\dfrac{x+3}{x-2},$

$x\neq -3,-2,2,3$

21. $\dfrac{x^3-8}{x^2-4}\cdot\dfrac{x+2}{3x}=\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\cdot\dfrac{x+2}{3x}$

$=\dfrac{x^2+2x+4}{3x}, x\neq -2,0,2$

22. $\dfrac{x^2+6x+9}{x^3+27}\cdot\dfrac{1}{x+3}$

$=\dfrac{(x+3)(x+3)}{(x+3)(x^2-3x+9)}\cdot\dfrac{1}{x+3}=\dfrac{1}{x^2-3x+9},$

$x\neq -3$

23. $\dfrac{x+1}{3} \div \dfrac{3x+3}{7} = \dfrac{x+1}{3} \div \dfrac{3(x+1)}{7}$

$= \dfrac{x+1}{3} \cdot \dfrac{7}{3(x+1)}$

$= \dfrac{7}{9}, x \neq -1$

24. $\dfrac{x+5}{7} \div \dfrac{4x+20}{9} = \dfrac{x+5}{7} \div \dfrac{4(x+5)}{9}$

$= \dfrac{x+5}{7} \cdot \dfrac{9}{4(x+5)}$

$= \dfrac{9}{28},$

$x \neq -5$

25. $\dfrac{x^2-4}{x} \div \dfrac{x+2}{x-2} = \dfrac{(x-2)(x+2)}{x} \cdot \dfrac{x-2}{x+2}$

$= \dfrac{(x-2)^2}{x}; x \neq 0, -2, 2$

26. $\dfrac{x^2-4}{x-2} \div \dfrac{x+2}{4x-8} = \dfrac{(x-2)(x+2)}{x-2} \div \dfrac{x+2}{4(x-2)}$

$= \dfrac{(x-2)(x+2)}{x-2} \cdot \dfrac{4(x-2)}{x+2}$

$= 4(x-2),$

$x \neq 2, -2$

27. $\dfrac{4x^2+10}{x-3} \div \dfrac{6x^2+15}{x^2-9}$

$= \dfrac{2(2x^2+5)}{x-3} \div \dfrac{3(2x^2+5)}{(x-3)(x+3)}$

$= \dfrac{2(2x^2+5)}{x-3} \cdot \dfrac{(x-3)(x+3)}{3(2x^2+5)}$

$= \dfrac{2(x+3)}{3}, x \neq 3, -3$

28. $\dfrac{x^2+x}{x^2-4} \div \dfrac{x^2-1}{x^2+5x+6}$

$= \dfrac{x(x+1)}{(x-2)(x+2)} \div \dfrac{(x-1)(x+1)}{(x+2)(x+3)}$

$= \dfrac{x(x+1)}{(x-2)(x+2)} \cdot \dfrac{(x+2)(x+3)}{(x-1)(x+1)}$

$= \dfrac{x(x+3)}{(x-2)(x-1)},$

$x \neq 2, 1, -1, -2, -3$

29. $\dfrac{x^2-25}{2x-2} \div \dfrac{x^2+10x+25}{x^2+4x-5}$

$= \dfrac{(x-5)(x+5)}{2(x-1)} \div \dfrac{(x+5)^2}{(x+5)(x-1)}$

$= \dfrac{(x-5)(x+5)}{2(x-1)} \cdot \dfrac{(x+5)(x-1)}{(x+5)^2}$

$= \dfrac{x-5}{2}, x \neq 1, -5$

30. $\dfrac{x^2-4}{x^2+3x-10} \div \dfrac{x^2+5x+6}{x^2+8x+15}$

$= \dfrac{(x+2)(x-2)}{(x+5)(x-2)} \div \dfrac{(x+2)(x+3)}{(x+3)(x+5)}$

$= \dfrac{(x+2)(x-2)}{(x+5)(x-2)} \cdot \dfrac{(x+3)(x+5)}{(x+2)(x+3)}$

$= 1$

$x \neq 2, -2, -3, -5$

31. $\dfrac{x^2+x-12}{x^2+x-30} \cdot \dfrac{x^2+5x+6}{x^2-2x-3} \div \dfrac{x+3}{x^2+7x+6}$

$= \dfrac{(x+4)(x-3)}{(x+6)(x-5)} \cdot \dfrac{(x+2)(x+3)}{(x+1)(x-3)} \cdot \dfrac{(x+6)(x+1)}{x+3}$

$= \dfrac{(x+4)(x+2)}{x-5}$

$x \neq -6, -3, -1, 3, 5$

32. $\dfrac{x^3-25x}{4x^2} \cdot \dfrac{2x^2-2}{x^2-6x+5} \div \dfrac{x^2+5x}{7x+7}$

$= \dfrac{x(x-5)(x+5)}{4x^2} \cdot \dfrac{2(x-1)(x+1)}{(x-1)(x-5)} \cdot \dfrac{7(x+1)}{x(x+5)}$

$= \dfrac{7(x+1)^2}{2x^2}$

$x \neq 0, 1, -1, 5, -5$

33. $\dfrac{4x+1}{6x+5} + \dfrac{8x+9}{6x+5} = \dfrac{4x+1+8x+9}{6x+5}$

$= \dfrac{12x+10}{6x+5}$

$= \dfrac{2(6x+5)}{6x+5} = 2, x \neq -\dfrac{5}{6}$

34. $\dfrac{3x+2}{3x+4}+\dfrac{3x+6}{3x+4}=\dfrac{3x+2+3x+6}{3x+4}$

$=\dfrac{6x+8}{3x+4}$

$=\dfrac{2(3x+4)}{3x+4}$

$=2$

$x\neq -\dfrac{4}{3}$

35. $\dfrac{x^2-2x}{x^2+3x}+\dfrac{x^2+x}{x^2+3x}=\dfrac{x^2-2x+x^2+x}{x^2+3x}$

$=\dfrac{2x^2-x}{x^2+3x}$

$=\dfrac{x(2x-1)}{x(x+3)}$

$=\dfrac{2x-1}{x+3},\ x\neq 0,-3$

36. $\dfrac{x^2-4x}{x^2-x-6}+\dfrac{4x-4}{x^2-x-6}=\dfrac{x^2-4x+4x-4}{x^2-x-6}$

$=\dfrac{x^2-4}{(x-3)(x+2)}$

$=\dfrac{(x-2)(x+2)}{(x-3)(x+2)}$

$=\dfrac{x-2}{x-3},$

$x\neq -2,\ 3$

37. $\dfrac{4x-10}{x-2}-\dfrac{x-4}{x-2}=\dfrac{4x-10-(x-4)}{x-2}$

$=\dfrac{4x-10-x+4}{x-2}$

$=\dfrac{3x-6}{x-2}$

$=\dfrac{3(x-2)}{x-2}$

$=3,\ x\neq 2$

38. $\dfrac{2x+3}{3x-6}-\dfrac{3-x}{3x-6}=\dfrac{2x+3-(3-x)}{3x-6}$

$=\dfrac{2x+3-3+x}{3x-6}$

$=\dfrac{3x}{3(x-2)}$

$=\dfrac{x}{x-2},$

$x\neq 2$

39. $\dfrac{x^2+3x}{x^2+x-12}-\dfrac{x^2-12}{x^2+x-12}$

$=\dfrac{x^2+3x-(x^2-12)}{x^2+x-12}$

$=\dfrac{x^2+3x-x^2+12}{x^2+x-12}$

$=\dfrac{3x+12}{x^2+x-12}$

$=\dfrac{3(x+4)}{(x+4)(x-3)}$

$=\dfrac{3}{x-3},\ x\neq 3,-4$

40. $\dfrac{x^2-4x}{x^2-x-6}-\dfrac{x-6}{x^2-x-6}$

$=\dfrac{x^2-4x-(x-6)}{x^2-x-6}$

$=\dfrac{x^2-4x-x+6}{x^2-x-6}$

$=\dfrac{x^2-5x+6}{x^2-x-6}$

$=\dfrac{(x-2)(x-3)}{(x-3)(x+2)}$

$=\dfrac{x-2}{x+2},\ x\neq -2,\ 3$

41. $\dfrac{3}{x+4}+\dfrac{6}{x+5}=\dfrac{3(x+5)+6(x+4)}{(x+4)(x+5)}$

$=\dfrac{3x+15+6x+24}{(x+4)(x+5)}$

$=\dfrac{9x+39}{(x+4)(x+5)},\ x\neq -4,\ -5$

42. $\dfrac{8}{x-2}+\dfrac{2}{x-3}=\dfrac{8(x-3)+2(x-2)}{(x-2)(x-3)}$

$=\dfrac{8x-24+2x-4}{(x-2)(x-3)}$

$=\dfrac{10x-28}{(x-2)(x-3)},$

$x\neq 2,\ 3$

43. $\dfrac{3}{x+1}-\dfrac{3}{x}=\dfrac{3x-3(x+1)}{x(x+1)}$

$=\dfrac{3x-3x-3}{x(x+1)}=-\dfrac{3}{x(x+1)}$, $x\neq -1, 0$

44. $\dfrac{4}{x}-\dfrac{3}{x+3}=\dfrac{4(x+3)-3x}{x(x+3)}$

$=\dfrac{4x+12-3x}{x(x+3)}$

$=\dfrac{x+12}{x(x+3)}$

$x\neq -3, 0$

45. $\dfrac{2x}{x+2}+\dfrac{x+2}{x-2}=\dfrac{2x(x-2)+(x+2)(x+2)}{(x+2)(x-2)}$

$=\dfrac{2x^2-4x+x^2+4x+4}{(x+2)(x-2)}$

$=\dfrac{3x^2+4}{(x+2)(x-2)}$, $x\neq -2, 2$

46. $\dfrac{3x}{x-3}-\dfrac{x+4}{x+2}=\dfrac{3x(x+2)-(x+4)(x-3)}{(x-3)(x+2)}$

$=\dfrac{3x^2+6x-\left(x^2+x-12\right)}{(x-3)(x+2)}$

$=\dfrac{2x^2+5x+12}{(x-3)(x+2)}$,

$x\neq 3, -2$

47. $\dfrac{x+5}{x-5}+\dfrac{x-5}{x+5}$

$=\dfrac{(x+5)(x+5)+(x-5)(x-5)}{(x-5)(x+5)}$

$=\dfrac{x^2+10x+25+x^2-10x+25}{(x-5)(x+5)}$

$=\dfrac{2x^2+50}{(x-5)(x+5)}$, $x\neq -5, 5$

48. $\dfrac{x+3}{x-3}+\dfrac{x-3}{x+3}=\dfrac{(x+3)(x+3)+(x-3)(x-3)}{(x-3)(x+3)}$

$=\dfrac{x^2+6x+9+x^2-6x+9}{(x-3)(x+3)}$

$=\dfrac{2x^2+18}{(x-3)(x+3)}$,

$x\neq -3, 3$

49. $\dfrac{3}{2x+4}+\dfrac{2}{3x+6}=\dfrac{3}{2(x+2)}+\dfrac{2}{3(x+2)}$

$=\dfrac{9}{6(x+2)}+\dfrac{4}{6(x+2)}$

$=\dfrac{9+4}{6(x+2)}$

$=\dfrac{13}{6(x+2)}$

$x\neq -2$

50. $\dfrac{5}{2x+8}+\dfrac{7}{3x+12}=\dfrac{5}{2(x+4)}+\dfrac{7}{3(x+4)}$

$=\dfrac{15}{6(x+4)}+\dfrac{14}{6(x+4)}$

$=\dfrac{15+14}{6(x+4)}$

$=\dfrac{29}{6(x+4)}$

$x\neq -4$

51. $\dfrac{4}{x^2+6x+9}+\dfrac{4}{x+3}=\dfrac{4}{(x+3)^2}+\dfrac{4}{x+3}$

$=\dfrac{4+4(x+3)}{(x+3)^2}=\dfrac{4+4x+12}{(x+3)^2}=\dfrac{4x+16}{(x+3)^2}$,

$x\neq -3$

52. $\dfrac{3}{5x+2}+\dfrac{5x}{25x^2-4}=\dfrac{3}{5x+2}+\dfrac{5x}{(5x-2)(5x+2)}$

$=\dfrac{3(5x-2)+5x}{(5x-2)(5x+2)}$

$=\dfrac{15x-6+5x}{(5x-2)(5x+2)}$

$=\dfrac{20x-6}{(5x-2)(5x+2)}$,

$x\neq -\dfrac{2}{5}, \dfrac{2}{5}$

53. $\dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$

$=\dfrac{3x}{(x+5)(x-2)}-\dfrac{2x}{(x+3)(x-2)}$

$=\dfrac{3x(x+3)-2x(x+5)}{(x+5)(x-2)(x+3)}$

$=\dfrac{3x^2+9x-2x^2-10x}{(x+5)(x-2)(x+3)}$

$=\dfrac{x^2-x}{(x+5)(x-2)(x+3)}$, $x\neq -5, 2, -3$

54. $\dfrac{x}{x^2-2x-24}-\dfrac{x}{x^2-7x+6}$

$=\dfrac{x}{(x-6)(x+4)}-\dfrac{x}{(x-6)(x-1)}$

$=\dfrac{x(x-1)-x(x+4)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2-x-x^2-4x}{(x-6)(x+4)(x-1)}$

$=-\dfrac{5x}{(x-6)(x-1)(x+4)}$,

$x\neq 6, 1, -4$

55. $\dfrac{x+3}{x^2-1}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{(x+1)(x+2)}{(x+1)(x-1)}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x^2+3x+2}{(x+1)(x-1)}$

$=\dfrac{x+3-x^2-3x-2}{(x+1)(x-1)}$

$=\dfrac{-x^2-2x+1}{(x+1)(x-1)}$

$x\neq 1, -1$

56. $\dfrac{x+5}{x^2-4}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{(x+2)(x+1)}{(x+2)(x-2)}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x^2+3x+2}{(x+2)(x-2)}$

$=\dfrac{x+5-x^2-3x-2}{(x+2)(x-2)}$

$=\dfrac{-x^2-2x+3}{(x+2)(x-2)}$

$x\neq 2, -2$

57. $\dfrac{4x^2+x-6}{x^2+3x+2}-\dfrac{3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-6}{(x+1)(x+2)}+\dfrac{-3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-5}{(x+1)(x+2)}+\dfrac{-3x(x+2)}{(x+1)(x+2)}+\dfrac{5(x+1)}{(x+1)(x+2)}$

$=\dfrac{4x^2+x-6-3x^2-6x+5x+5}{(x+1)(x+2)}$

$=\dfrac{x^2-1}{(x+1)(x+2)}$

$=\dfrac{(x-1)(x+1)}{(x+1)(x+2)}$

$=\dfrac{x-1}{x+2}; x\neq -2, -1$

58. $\dfrac{6x^2+17x-40}{x^2+x-20}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40}{(x+5)(x-4)}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40+3(x+5)-5x(x-4)}{(x+5)(x-4)}$

$=\dfrac{6x^2+17x-40+3x+15-5x^2+20x}{(x+5)(x-4)}$

$=\dfrac{x^2+40x-25}{(x+5)(x-4)}; x\neq -5, 4$

59. $\dfrac{\frac{x}{3}-1}{x-3}=\dfrac{3\left[\frac{x}{3}-1\right]}{3[x-3]}=\dfrac{x-3}{3(x-3)}=\dfrac{1}{3},\ x\neq 3$

60. $\dfrac{\frac{x}{4}-1}{x-4}=\dfrac{4\left[\frac{x}{4}-1\right]}{4(x-4)}=\dfrac{x-4}{4(x-4)}=\dfrac{1}{4},\ x\neq 4$

61. $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}=\dfrac{x\left[1+\frac{1}{x}\right]}{x\left[3-\frac{1}{x}\right]}=\dfrac{x+1}{3x-1},\ x\neq 0,\dfrac{1}{3}$

62. $\dfrac{8+\frac{1}{x}}{4-\frac{1}{x}}=\dfrac{x\left[8+\frac{1}{x}\right]}{x\left[4-\frac{1}{x}\right]}=\dfrac{8x+1}{4x-1},\ x\neq 0,\dfrac{1}{4}$

63. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y}=\dfrac{xy\left[\frac{1}{x}+\frac{1}{y}\right]}{xy[x+y]}=\dfrac{y+x}{xy(x+y)}=\dfrac{1}{xy},$
$x\neq 0,\ y\neq 0,\ x\neq -y$

64. $\dfrac{1-\frac{1}{x}}{xy}=\dfrac{x\left[1-\frac{1}{x}\right]}{x(xy)}=\dfrac{x-1}{x^2y},\ x\neq 0,\ y\neq 0$

65. $\dfrac{x-\frac{x}{x+3}}{x+2}=\dfrac{(x+3)\left[x-\frac{x}{x+3}\right]}{(x+3)(x+2)}=\dfrac{x(x+3)-x}{(x+3)(x+2)}$
$=\dfrac{x^2+3x-x}{(x+3)(x+2)}=\dfrac{x^2+2x}{(x+3)(x+2)}$
$=\dfrac{x(x+2)}{(x+3)(x+2)}=\dfrac{x}{x+3},\ x\neq -2,-3$

66. $\dfrac{x-3}{x-\frac{3}{x-2}}=\dfrac{(x-2)[x-3]}{(x-2)\left[x-\frac{3}{x-2}\right]}=\dfrac{(x-2)(x-3)}{x(x-2)-3}$
$=\dfrac{(x-2)(x-3)}{x^2-2x-3}$
$=\dfrac{(x-2)(x-3)}{(x-3)(x+1)}=\dfrac{x-2}{x+1},\ x\neq 2,3,-1$

67. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}=\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{(x-2)(x+2)}}$
$=\dfrac{\left[\frac{3}{x-2}-\frac{4}{x+2}\right](x-2)(x+2)}{\left[\frac{7}{(x-2)(x+2)}\right](x-2)(x+2)}$
$=\dfrac{3(x+2)-4(x-2)}{7}$
$=\dfrac{3x+6-4x+8}{7}=\dfrac{-x+14}{7}$
$=-\dfrac{x-14}{7}\quad x\neq -2,\ 2$

68. $\dfrac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}=\dfrac{\frac{x}{x-2}+1}{\frac{3}{(x-2)(x+2)}+1}$
$=\dfrac{\left[\frac{x}{x-2}+1\right](x-2)(x+2)}{\left[\frac{3}{(x-2)(x+2)}+1\right](x-2)(x+2}$
$=\dfrac{x(x+2)+(x-2)(x+2)}{3+(x-2)(x+2)}$
$=\dfrac{x^2+2x+x^2-4}{3+x^2-4}=\dfrac{2x^2+2x-4}{x^2-1}$
$=\dfrac{2(x^2+x-2)}{(x-1)(x+1)}$
$=\dfrac{2(x+2)(x-1)}{(x-1)(x+1)}=\dfrac{2(x+2)}{x+1},$
$x\neq 1,-1,\ 2,\ -2$

69. $\dfrac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3}+\frac{1}{x-3}}=\dfrac{\frac{1}{x+1}}{\frac{1}{(x+1)(x-3)}+\frac{1}{x-3}}$
$=\dfrac{\frac{(x+1)(x-3)}{x+1}}{\frac{(x+1)(x-3)}{(x+1)(x-3)}+\frac{(x+1)(x-3)}{x-3}}$
$=\dfrac{x-3}{1+x+1}$
$=\dfrac{x-3}{x+2}\quad x\neq -2,-1,3$

70.
$$\frac{\dfrac{6}{x^2+2x-15}-\dfrac{1}{x-3}}{\dfrac{1}{x+5}+1}=\frac{\dfrac{6}{(x+5)(x-3)}-\dfrac{1}{x-3}}{\dfrac{1}{x+5}+1}$$
$$=\frac{\dfrac{6(x+5)(x-3)}{(x+5)(x-3)}-\dfrac{(x+5)(x-3)}{x-3}}{\dfrac{(x+5)(x-3)}{x+5}+(x+5)(x-3)}$$
$$=\frac{6-(x+5)}{(x-3)+(x+5)(x-3)}$$
$$=\frac{6-x-5}{x-3+x^2+2x-15}$$
$$=\frac{1-x}{x^2+3x-18}$$
$$=\frac{1-x}{(x+6)(x-3)} \quad x\neq -6,-5,3$$

71.
$$\frac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\frac{\dfrac{x^2(x+h)^2}{(x+h)^2}-\dfrac{x^2(x+h)^2}{x^2}}{hx^2(x+h)^2}$$
$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$
$$=\frac{x^2-(x^2+2hx+h^2)}{hx^2(x+h)^2}$$
$$=\frac{x^2-x^2-2hx-h^2}{hx^2(x+h)^2}$$
$$=\frac{-2hx-h^2}{hx^2(x+h)^2}$$
$$=\frac{-h(2x+h)}{hx^2(x+h)^2}$$
$$=-\frac{(2x+h)}{x^2(x+h)^2}$$

72.
$$\frac{\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}}{h}=\frac{\dfrac{(x+h)(x+h+1)(x+1)}{x+h+1}-\dfrac{x(x+h+1)(x+1)}{x+1}}{h(x+h+1)(x+1)}$$
$$=\frac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)}$$
$$=\frac{x^2+x+hx+h-x^2-hx-x}{h(x+h+1)(x+1)}$$
$$=\frac{h}{h(x+h+1)(x+1)}$$
$$=\frac{1}{(x+h+1)(x+1)}$$

73. $\left(\dfrac{2x+3}{x+1}\cdot\dfrac{x^2+4x-5}{2x^2+x-3}\right)-\dfrac{2}{x+2}=\left(\dfrac{\cancel{(2x+3)}}{x+1}\cdot\dfrac{(x+5)\cancel{(x-1)}}{\cancel{(2x+3)}\cancel{(x-1)}}\right)-\dfrac{2}{x+2}=\dfrac{x+5}{x+1}-\dfrac{2}{x+2}$

$=\dfrac{(x+5)(x+2)}{(x+1)(x+2)}-\dfrac{2(x+1)}{(x+1)(x+2)}=\dfrac{(x+5)(x+2)-2(x+1)}{(x+1)(x+2)}=\dfrac{x^2+2x+5x+10-2x-2}{(x+1)(x+2)}=\dfrac{x^2+5x+8}{(x+1)(x+2)}$

74. $\dfrac{1}{x^2-2x-8}\cdot\left(\dfrac{1}{x-4}-\dfrac{1}{x+2}\right)=\dfrac{1}{(x-4)(x+2)}\div\left(\dfrac{(x+2)}{(x-4)(x+2)}-\dfrac{(x-4)}{(x-4)(x+2)}\right)$

$=\dfrac{1}{(x-4)(x+2)}\div\left(\dfrac{x+2-x+4}{(x-4)(x+2)}\right)=\dfrac{1}{(x-4)(x+2)}\div\left(\dfrac{6}{(x-4)(x+2)}\right)=\dfrac{1}{(x-4)(x+2)}\cdot\dfrac{(x-4)(x+2)}{6}=\dfrac{1}{6}$

75. $\left(2-\dfrac{6}{x+1}\right)\left(1+\dfrac{3}{x-2}\right)=\left(\dfrac{2(x+1)}{(x+1)}-\dfrac{6}{(x+1)}\right)\left(\dfrac{(x-2)}{(x-2)}+\dfrac{3}{(x-2)}\right)$

$=\left(\dfrac{2x+2-6}{x+1}\right)\left(\dfrac{x-2+3}{x-2}\right)=\left(\dfrac{2x-4}{x+1}\right)\left(\dfrac{x+1}{x-2}\right)=\dfrac{2\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-2)}}=2$

76. $\left(4-\dfrac{3}{x+2}\right)\left(1+\dfrac{5}{x-1}\right)=\left(\dfrac{4(x+2)}{x+2}-\dfrac{3}{x+2}\right)\left(\dfrac{(x-1)}{x-1}+\dfrac{5}{x-1}\right)$

$=\left(\dfrac{4x+8-3}{x+2}\right)\left(\dfrac{x-1+5}{x-1}\right)=\dfrac{4x+5}{x+2}\cdot\dfrac{x+4}{x-1}=\dfrac{(4x+5)(x+4)}{(x+2)(x-1)}$

77. $\dfrac{y^{-1}-(y+5)^{-1}}{5}=\dfrac{\dfrac{1}{y}-\dfrac{1}{y+5}}{5}$

$\text{LCD}=y(y+5)$

$\dfrac{\dfrac{1}{y}-\dfrac{1}{y+5}}{5}=\dfrac{y(y+5)\left(\dfrac{1}{y}-\dfrac{1}{y+5}\right)}{y(y+5)(5)}=\dfrac{y+5-y}{5y(y+5)}=\dfrac{5}{5y(y+5)}=\dfrac{1}{y(y+5)}$

78. $\dfrac{y^{-1}-(y+2)^{-1}}{2}=\dfrac{\dfrac{1}{y}-\dfrac{1}{y+2}}{2}$

$\text{LCD}=y(y+2)$

$\dfrac{\dfrac{1}{y}-\dfrac{1}{y+2}}{2}=\dfrac{y(y+2)\left(\dfrac{1}{y}-\dfrac{1}{y+2}\right)}{y(y+2)(2)}=\dfrac{y+2-y}{2y(y+2)}=\dfrac{2}{2y(y+2)}=\dfrac{1}{y(y+2)}$

79. $\left(\dfrac{1}{a^3-b^3}\cdot\dfrac{ac+ad-bc-bd}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}=\left(\dfrac{1}{(a-b)\left(a^2+ab+b^2\right)}\cdot\dfrac{a(c+d)-b(c+d)}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}$

$=\left(\dfrac{1}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\cdot\dfrac{(c+d)\cancel{(a-b)}}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}=\dfrac{c+d}{a^2+ab+b^2}-\dfrac{c-d}{a^2+bd+b^2}$

$=\dfrac{c+d-c+d}{a^2+ab+b^2}=\dfrac{2d}{a^2+ab+b^2}$

80. $\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\dfrac{a^3-b^3}{a^3+b^3}\right)=\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\dfrac{a^3+b^3}{a^3-b^3}\right)$

$=\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\dfrac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right)=\dfrac{ab}{a^2+ab+b^2}+\dfrac{a^2-ab+b^2}{a^2+ab+b^2}$

$=\dfrac{ab+a^2-ab+b^2}{a^2+ab+b^2}=\dfrac{a^2+b^2}{a^2+ab+b^2}$

81. **a.** $\dfrac{130x}{100-x}$ is equal to

1. $\dfrac{130\cdot40}{100-40}=\dfrac{130\cdot40}{60}=86.67$, when $x=40$
2. $\dfrac{130\cdot80}{100-80}=\dfrac{130\cdot80}{20}=520$, when $x=80$
3. $\dfrac{130\cdot90}{100-90}=\dfrac{130\cdot90}{10}=1170$, when $x=90$

It costs \$86,670,000 to inoculate 40% of the population against this strain of flu, and \$520,000,000 to inoculate 80% of the population, and \$1,170,000,000 to inoculate 90% of the population.

b. For $x=100$, the function is not defined.

c. As x approaches 100, the value of the function increases rapidly. So it costs an astronomical amount of money to inoculate almost all of the people, and it is impossible to inoculate 100% of the population.

82. $\dfrac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}}$

LCD = $r_1 r_2$

$$\frac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}} = \frac{r_1 r_2 (2d)}{r_1 r_2\left(\dfrac{d}{r_1}+\dfrac{d}{r_2}\right)}$$

$$= \frac{2r_1 r_2 d}{r_2 d + r_1 d}$$

$$= \frac{2r_1 r_2 d}{d(r_2 + r_1)} = \frac{2r_1 r_2}{r_2 + r_1}$$

If $r_1 = 40$ and $r_2 = 30$, the value of this expression will be

$$\frac{2 \cdot 40 \cdot 30}{30+40} = \frac{2400}{70}$$

$$= 34\frac{2}{7}.$$

Your average speed will be $34\dfrac{2}{7}$ miles per hour.

83. **a.** Substitute 4 for x in the model.

$W = -66x^2 + 526x + 1030$

$W = -66(4)^2 + 526(4) + 1030$

$W = 2078$

According to the model, women between the ages of 19 and 30 with this lifestyle need 2078 calories per day. This underestimates the actual value shown in the bar graph by 22 calories.

b. Substitute 4 for x in the model.

$M = -120x^2 + 998x + 590$

$M = -120(4)^2 + 998(4) + 590$

$M = 2662$

According to the model, men between the ages of 19 and 30 with this lifestyle need 2662 calories per day. This underestimates the actual value shown in the bar graph by 38 calories.

c. $$\frac{W}{M} = \frac{-66x^2 + 526x + 1030}{-120x^2 + 998x + 590}$$

$$= \frac{2(-33x^2 + 263x + 515)}{2(-60x^2 + 499x + 295)}$$

$$= \frac{-33x^2 + 263x + 515}{-60x^2 + 499x + 295}$$

84. $P = 2L + 2W$

$$= 2\left(\frac{x}{x+3}\right) + 2\left(\frac{x}{x-4}\right)$$

$$= \frac{2x}{x+3} + \frac{2x}{x+4}$$

$$= \frac{2x(x+4)}{(x+3)(x+4)} + \frac{2x(x+3)}{(x+3)(x+4)}$$

$$= \frac{2x^2 + 8x + 2x^2 + 6x}{(x+3)(x+4)}$$

$$= \frac{4x^2 + 14x}{(x+3)(x+4)}$$

85. $P = 2L + 2W$

$$= 2\left(\frac{x}{x+5}\right) + 2\left(\frac{x}{x+6}\right)$$

$$= \frac{2x}{x+5} + \frac{2x}{x+6}$$

$$= \frac{2x(x+6)}{(x+5)(x+6)} + \frac{2x(x+5)}{(x+5)(x+6)}$$

$$= \frac{2x^2 + 12x + 2x^2 + 10x}{(x+5)(x+6)}$$

$$= \frac{4x^2 + 22x}{(x+5)(x+6)}$$

86. – 97. Answers will vary.

98. does not make sense; Explanations will vary. Sample explanation: $\frac{3x-3}{4x(x-1)} = \frac{3(1)-3}{4(1)(1-1)} = \frac{0}{0}$ which is undefined.

99. does not make sense; Explanations will vary. Sample explanation: The numerator and denominator of $\frac{7}{14+x}$ do not share a common factor.

100. does not make sense; Explanations will vary. Sample explanation: The first step is to invert the second fraction.

101. makes sense

102. false; Changes to make the statement true will vary. A sample change is: $\frac{x^2-25}{x-5} = \frac{(x+5)(x-5)}{x-5} = x+5$

103. true

104. true

105. false; Changes to make the statement true will vary. A sample change is: $6 + \frac{1}{x} = \frac{6x}{x} + \frac{1}{x} = \frac{6x+1}{x}$

106. $\dfrac{1}{x^n-1}-\dfrac{1}{x^n+1}-\dfrac{1}{x^{2n}-1}$

$=\dfrac{x^n+1}{x^{2n}-1}-\dfrac{x^n-1}{x^{2n}-1}-\dfrac{1}{x^{2n}-1}$

$=\dfrac{x^n+1-x^n+1-1}{x^{2n}-1}$

$=\dfrac{1}{x^{2n}-1}$

107. $\left(1-\dfrac{1}{x}\right)\left(1-\dfrac{1}{x+1}\right)\left(1-\dfrac{1}{x+2}\right)\left(1-\dfrac{1}{x+3}\right)=\left(\dfrac{x}{x}-\dfrac{1}{x}\right)\left(\dfrac{x+1}{x+1}-\dfrac{1}{x+1}\right)\left(\dfrac{x+2}{x+2}-\dfrac{1}{x+2}\right)\left(\dfrac{x+3}{x+3}-\dfrac{1}{x+3}\right)$

$=\left(\dfrac{x-1}{x}\right)\left(\dfrac{(x+1)-1}{x+1}\right)\left(\dfrac{(x+2)-1}{x+2}\right)\left(\dfrac{(x+3)-1}{x+3}\right)$

$=\dfrac{x-1}{\cancel{x}}\cdot\dfrac{\cancel{x}}{\cancel{x+1}}\cdot\dfrac{\cancel{x+1}}{\cancel{x+2}}\cdot\dfrac{\cancel{x+2}}{x+3}=\dfrac{x-1}{x+3}$

108. $(x-y)^{-1}+(x-y)^{-2}=\dfrac{1}{(x-y)}+\dfrac{1}{(x-y)^2}=\dfrac{(x-y)}{(x-y)(x-y)}+\dfrac{1}{(x-y)^2}=\dfrac{x-y+1}{(x-y)^2}$

109. It cubes x.

$$\frac{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{\frac{1}{x^4}+\frac{1}{x^5}+\frac{1}{x^6}}=\frac{\frac{x^6}{x}+\frac{x^6}{x^2}+\frac{x^6}{x^3}}{\frac{x^6}{x^4}+\frac{x^6}{x^5}+\frac{x^6}{x^6}}=\frac{x^5+x^4+x^3}{x^2+x+1}=\frac{x^3\left(x^2+x+1\right)}{x^2+x+1}=x^3$$

110. $y=4-x^2$

111. $y=1-x^2$

112. $y=|x+1|$

x	$y=\lvert x+1\rvert$
-4	$\lvert-4+1\rvert=3$
-3	$\lvert-3+1\rvert=2$
-2	$\lvert-2+1\rvert=1$
-1	$\lvert-1+1\rvert=0$
0	$\lvert0+1\rvert=1$
1	$\lvert1+1\rvert=2$
2	$\lvert2+1\rvert=3$

Chapter P Review Exercises

1. $3+6(x-2)^3 = 3+6(4-2)^3$
$= 3+6(2)^3$
$= 3+6(8)$
$= 3+48$
$= 51$

2. $x^2-5(x-y) = 6^2-5(6-2)$
$= 36-5(4)$
$= 36-20$
$= 16$

3. $S = 0.015x^2 + x + 10$
$S = 0.015(60)^2 + (60) + 10$
$= 0.015(3600) + 60 + 10$
$= 54 + 60 + 10$
$= 124$

4. $A = \{a,b,c\} \quad B = \{a,c,d,e\}$
$\{a,b,c\} \cap \{a,c,d,e\} = \{a,c\}$

5. $A = \{a,b,c\} \quad B = \{a,c,d,e\}$
$\{a,b,c\} \cup \{a,c,d,e\} = \{a,b,c,d,e\}$

6. $A = \{a,b,c\} \quad C = \{a,d,f,g\}$
$\{a,b,c\} \cup \{a,d,f,g\} = \{a,b,c,d,f,g\}$

7. $A = \{a,b,c\} \quad C = \{a,d,f,g\}$
$\{a,d,f,g\} \cap \{a,b,c\} = \{a\}$

8. **a.** $\sqrt{81}$

b. $0, \sqrt{81}$

c. $-17, 0, \sqrt{81}$

d. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$

e. $\sqrt{2}, \pi$

f. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$

9. $|-103| = 103$

10. $|\sqrt{2}-1| = \sqrt{2}-1$

11. $|3-\sqrt{17}| = \sqrt{17}-3$ since $\sqrt{17}$ is greater than 3.

12. $|4-(-17)| = |4+17| = |21| = 21$

13. $3 + 17 = 17 + 3$;
commutative property of addition.

14. $(6\cdot 3)\cdot 9 = 6\cdot(3\cdot 9)$;
associative property of multiplication.

15. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = \sqrt{15}+3$;
distributive property of multiplication over addition.

16. $(6\cdot 9)\cdot 2 = 2\cdot(6\cdot 9)$;
commutative property of multiplication.

17. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = (\sqrt{5}+\sqrt{3})\sqrt{3}$;
commutative property of multiplication.

18. $(3\cdot 7)+(4\cdot 7) = (4\cdot 7)+(3\cdot 7)$;
commutative property of addition.

19. $5(2x-3)+7x = 10x-15+7x = 17x-15$

20. $\frac{1}{5}(5x)+[(3y)+(-3y)]-(-x) = x+[0]+x = 2x$

21. $3(4y-5)-(7y+2) = 12y-15-7y-2 = 5y-17$

22. $8-2[3-(5x-1)] = 8-2[3-5x+1]$
$= 8-2[4-5x]$
$= 8-8+10x$
$= 10x$

23. $D = 0.005x^2 + 0.55x + 34$
$D = 0.005(30)^2 + 0.55(30) + 34$
$= 55$
The U.S. diversity index was 55% in 2010.
This is the same as the value displayed in the bar graph.

24. $(-3)^3(-2)^2 = (-27)\cdot(4) = -108$

25. $2^{-4}+4^{-1}=\frac{1}{2^4}+\frac{1}{4}$
$=\frac{1}{16}+\frac{1}{4}$
$=\frac{1}{16}+\frac{4}{16}$
$=\frac{5}{16}$

26. $5^{-3}\cdot 5=5^{-3}5^{1}=5^{-3+1}=5^{-2}=\frac{1}{5^2}=\frac{1}{25}$

27. $\frac{3^3}{3^6}=3^{3-6}=3^{-3}=\frac{1}{3^3}=\frac{1}{27}$

28. $(-2x^4y^3)^3=(-2)^3(x^4)^3(y^3)^3$
$=(-2)^3x^{4\cdot 3}y^{3\cdot 3}$
$=-8x^{12}y^9$

29. $(-5x^3y^2)(-2x^{-11}y^{-2})$
$=(-5)(-2)x^3x^{-11}y^2y^{-2}$
$=10\cdot x^{3-11}y^{2-2}$
$=10x^{-8}y^0$
$=\frac{10}{x^8}$

30. $(2x^3)^{-4}=(2)^{-4}(x^3)^{-4}$
$=2^{-4}x^{-12}$
$=\frac{1}{2^4x^{12}}$
$=\frac{1}{16x^{12}}$

31. $\frac{7x^5y^6}{28x^{15}y^{-2}}=\left(\frac{7}{28}\right)(x^{5-15})(y^{6-(-2)})$
$=\frac{1}{4}x^{-10}y^8$
$=\frac{y^8}{4x^{10}}$

32. $3.74\times 10^4=37,400$

33. $7.45\times 10^{-5}=0.0000745$

34. $3,590,000=3.59\times 10^6$

35. $0.00725=7.25\times 10^{-3}$

36. $(3\times 10^3)(1.3\times 10^2)=(3\times 1.3)\times(10^3\times 10^2)$
$=3.9\times 10^5$
$=390,000$

37. $\frac{6.9\times 10^3}{3\times 10^5}=\left(\frac{6.9}{3}\right)\times 10^{3-5}$
$=2.3\times 10^{-2}$
$=0.023$

38. 1.35×10^{12}

39. $32,000,000=3.2\times 10^7$

40. $\frac{1.35\times 10^{12}}{3.2\times 10^7}=\frac{1.35}{3.2}\cdot\frac{10^{12}}{10^7}\approx 0.42188\times 10^5=42,188$
1.35×10^{12} seconds is approximately 42,188 years.

41. $\sqrt{300}=\sqrt{100\cdot 3}=\sqrt{100}\cdot\sqrt{3}=10\sqrt{3}$

42. $\sqrt{12x^2}=\sqrt{4x^2\cdot 3}=\sqrt{4x^2}\cdot\sqrt{3}=2|x|\sqrt{3}$

43. $\sqrt{10x}\cdot\sqrt{2x}=\sqrt{20x^2}$
$=\sqrt{4x^2}\cdot\sqrt{5}$
$=2x\sqrt{5}$

44. $\sqrt{r^3}=\sqrt{r^2}\cdot\sqrt{r}=r\sqrt{r}$

45. $\sqrt{\frac{121}{4}}=\frac{\sqrt{121}}{\sqrt{4}}=\frac{11}{2}$

46. $\frac{\sqrt{96x^3}}{\sqrt{2x}}=\sqrt{\frac{96x^3}{2x}}$
$=\sqrt{48x^2}$
$=\sqrt{16x^2}\cdot\sqrt{3}$
$=4x\sqrt{3}$

47. $7\sqrt{5}+13\sqrt{5}=(7+13)\sqrt{5}=20\sqrt{5}$

48. $2\sqrt{50}+3\sqrt{8}=2\sqrt{25\cdot 2}+3\sqrt{4\cdot 2}$
$=2\cdot 5\sqrt{2}+3\cdot 2\sqrt{2}$
$=10\sqrt{2}+6\sqrt{2}$
$=16\sqrt{2}$

49. $4\sqrt{72}-2\sqrt{48}=4\sqrt{36\cdot 2}-2\sqrt{16\cdot 3}$
$=4\cdot 6\sqrt{2}-2\cdot 4\sqrt{3}$
$=24\sqrt{2}-8\sqrt{3}$

50. $\dfrac{30}{\sqrt{5}}=\dfrac{30}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{30\sqrt{5}}{5}=6\sqrt{5}$

51. $\dfrac{\sqrt{2}}{\sqrt{3}}=\dfrac{\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{6}}{3}$

52. $\dfrac{5}{6+\sqrt{3}}=\dfrac{5}{6+\sqrt{3}}\cdot\dfrac{6-\sqrt{3}}{6-\sqrt{3}}$
$=\dfrac{5(6-\sqrt{3})}{36-3}$
$=\dfrac{5(6-\sqrt{3})}{33}$

53. $\dfrac{14}{\sqrt{7}-\sqrt{5}}=\dfrac{14}{\sqrt{7}-\sqrt{5}}\cdot\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$
$=\dfrac{14(\sqrt{7}+\sqrt{5})}{7-5}$
$=\dfrac{14(\sqrt{7}+\sqrt{5})}{2}$
$=7(\sqrt{7}+\sqrt{5})$

54. $\sqrt[3]{125}=5$

55. $\sqrt[5]{-32}=-2$

56. $\sqrt[4]{-125}$ is not a real number.

57. $\sqrt[4]{(-5)^4}=\sqrt[4]{625}=\sqrt[4]{5^4}=5$

58. $\sqrt[3]{81}=\sqrt[3]{27\cdot 3}=\sqrt[3]{27}\cdot\sqrt[3]{3}=3\sqrt[3]{3}$

59. $\sqrt[3]{y^5}=\sqrt[3]{y^3y^2}=y\sqrt[3]{y^2}$

60. $\sqrt[4]{8}\cdot\sqrt[4]{10}=\sqrt[4]{80}=\sqrt[4]{16\cdot 5}=\sqrt[4]{16}\cdot\sqrt[4]{5}=2\sqrt[4]{5}$

61. $4\sqrt[3]{16}+5\sqrt[3]{2}=4\sqrt[3]{8\cdot 2}+5\sqrt[3]{2}$
$=4\cdot 2\sqrt[3]{2}+5\sqrt[3]{2}$
$=8\sqrt[3]{2}+5\sqrt[3]{2}$
$=13\sqrt[3]{2}$

62. $\dfrac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}=\sqrt[4]{\dfrac{32x^5}{16x}}=\sqrt[4]{2x^4}=x\sqrt[4]{2}$

63. $16^{1/2}=\sqrt{16}=4$

64. $25^{-1/2}=\dfrac{1}{25^{1/2}}=\dfrac{1}{\sqrt{25}}=\dfrac{1}{5}$

65. $125^{1/3}=\sqrt[3]{125}=5$

66. $27^{-1/3}=\dfrac{1}{27^{1/3}}=\dfrac{1}{\sqrt[3]{27}}=\dfrac{1}{3}$

67. $64^{2/3}=(\sqrt[3]{64})^2=4^2=16$

68. $27^{-4/3}=\dfrac{1}{27^{4/3}}=\dfrac{1}{(\sqrt[3]{27})^4}=\dfrac{1}{3^4}=\dfrac{1}{81}$

69. $(5x^{2/3})(4x^{1/4})=5\cdot 4x^{2/3+1/4}=20x^{11/12}$

70. $\dfrac{15x^{3/4}}{5x^{1/2}}=\left(\dfrac{15}{5}\right)x^{3/4-1/2}=3x^{1/4}$

71. $(125\cdot x^6)^{2/3}=(\sqrt[3]{125x^6})^2$
$=(5x^2)^2$
$=25x^4$

72. $\sqrt[6]{y^3}=(y^3)^{1/6}=y^{3\cdot 1/6}=y^{1/2}=\sqrt{y}$

73. $(-6x^3+7x^2-9x+3)+(14x^3+3x^2-11x-7)=(-6x^3+14x^3)+(7x^2+3x^2)+(-9x-11x)+(3-7)$
$=8x^3+10x^2-20x-4$

The degree is 3.

74. $(13x^4-8x^3+2x^2)-(5x^4-3x^3+2x^2-6)=(13x^4-8x^3+2x^2)+(-5x^4+3x^3-2x^2+6)$
$=(13x^4-5x^4)+(-8x^3+3x^3)+(2x^2-2x^2)+6$
$=8x^4-5x^3+6$

The degree is 4.

75. $(3x-2)(4x^2+3x-5)=(3x)(4x^2)+(3x)(3x)+(3x)(-5)+(-2)(4x^2)+(-2)(3x)+(-2)(-5)$
$=12x^3+9x^2-15x-8x^2-6x+10$
$=12x^3+x^2-21x+10$

76. $(3x-5)(2x+1)=(3x)(2x)+(3x)(1)+(-5)(2x)+(-5)(1)$
$=6x^2+3x-10x-5$
$=6x^2-7x-5$

77. $(4x+5)(4x-5)=(4x^2)-5^2=16x^2-25$

78. $(2x+5)^2=(2x)^2+2(2x)\cdot 5+5^2=4x^2+20x+25$

79. $(3x-4)^2=(3x)^2-2(3x)\cdot 4+(-4)^2=9x^2-24x+16$

80. $(2x+1)^3=(2x)^3+3(2x)^2(1)+3(2x)(1)^2+1^3=8x^3+12x^2+6x+1$

81. $(5x-2)^3=(5x)^3-3(5x)^2(2)+3(5x)(2)^2-2^3=125x^3-150x^2+60x-8$

82. $(7x^2-8xy+y^2)+(-8x^2-9xy-4y^2)=(7x^2-8x^2)+(-8xy-9xy)+(y^2-4y^2)$
$=-x^2-17xy-3y^2$

The degree is 2.

83. $(13x^3y^2-5x^2y-9x^2)-(-11x^3y^2-6x^2y+3x^2-4)$
$=(13x^3y^2-5x^2y-9x^2)+(11x^3y^2+6x^2y-3x^2+4)$
$=(13x^3y^2+11x^3y^2)+(-5x^2y+6x^2y)+(-9x^2-3x^2)+4$
$=24x^3y^2+x^2y-12x^2+4$

The degree is 5.

84. $(x+7y)(3x-5y)=x(3x)+(x)(-5y)+(7y)(3x)+(7y)(-5y)$
$=3x^2-5xy+21xy-35y^2$
$=3x^2+16xy-35y^2$

85. $(3x-5y)^2=(3x)^2-2(3x)(5y)+(-5y)^2$
$=9x^2-30xy+25y^2$

86. $(3x^2+2y)^2 = (3x^2)^2 + 2(3x^2)(2y) + (2y)^2$
$= 9x^4 + 12x^2y + 4y^2$

87. $(7x+4y)(7x-4y) = (7x)^2 - (4y)^2$
$= 49x^2 - 16y^2$

88. $(a-b)(a^2+ab+b^2)$
$= a(a^2) + a(ab) + a(b^2) + (-b)(a^2)$
$+(-b)(ab) + (-b)(b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

89. $15x^3 + 3x^2 = 3x^2 \cdot 5x + 3x^2 \cdot 1$
$= 3x^2(5x+1)$

90. $x^2 - 11x + 28 = (x-4)(x-7)$

91. $15x^2 - x - 2 = (3x+1)(5x-2)$

92. $64 - x^2 = 8^2 - x^2 = (8-x)(8+x)$

93. $x^2 + 16$ is prime.

94. $3x^4 - 9x^3 - 30x^2 = 3x^2(x^2 - 3x - 10)$
$= 3x^2(x-5)(x+2)$

95. $20x^7 - 36x^3 = 4x^3(5x^4 - 9)$

96. $x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3)$
$= (x^2-9)(x-3)$
$= (x+3)(x-3)(x-3)$
$= (x+3)(x-3)^2$

97. $16x^2 - 40x + 25 = (4x-5)(4x-5)$
$= (4x-5)^2$

98. $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2+4)(x^2-4)$
$= (x^2+4)(x+2)(x-2)$

99. $y^3 - 8 = y^3 - 2^3 = (y-2)(y^2+2y+4)$

100. $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2-4x+16)$

101. $3x^4 - 12x^2 = 3x^2(x^2-4)$
$= 3x^2(x-2)(x+2)$

102. $27x^3 - 125 = (3x)^3 - 5^3$
$= (3x-5)[(3x)^2 + (3x)(5) + 5^2]$
$= (3x-5)(9x^2 + 15x + 25)$

103. $x^5 - x = x(x^4 - 1)$
$= x(x^2-1)(x^2+1)$
$= x(x-1)(x+1)(x^2+1)$

104. $x^3 + 5x^2 - 2x - 10 = x^2(x+5) - 2(x+5)$
$= (x^2-2)(x+5)$

105. $x^2 + 18x + 81 - y^2 = \left(x^2 + 18x + 81\right) - y^2$
$= \left(x+9\right)^2 - y^2$
$= \left(x+9-y\right)\left(x+9+y\right)$

106. $16x^{-3/4} + 32x^{1/4} = 16x^{-3/4}\left(1 + 2x^{1/4-(-3/4)}\right)$
$= 16x^{-3/4}\left(1+2x\right)$
$= \dfrac{16\left(1+2x\right)}{x^{3/4}}$

107. $\left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}} - \left(x^2-4\right)^2\left(x^2+3\right)^{\frac{3}{2}}$
$= \left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x^2-4\right)\left(x^2+3\right)\right]$
$= \left(x-2\right)\left(x+2\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x-2\right)\left(x+2\right)\left(x^2+3\right)\right]$
$= (x-2)(x+2)(x^2+3)^{\frac{1}{2}}(-x^4 + x^2 + 13)$

108. $12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}} = 6x^{-\frac{3}{2}}\left(2x+1\right) = \dfrac{6(2x+1)}{x^{\frac{3}{2}}}$

109. $\dfrac{x^3+2x^2}{x+2} = \dfrac{x^2(x+2)}{x+2} = x^2$, $x \neq -2$

110. $\dfrac{x^2+3x-18}{x^2-36} = \dfrac{(x+6)(x-3)}{(x+6)(x-6)} = \dfrac{x-3}{x-6}$,
$x \neq -6, 6$

111. $\dfrac{x^2+2x}{x^2+4x+4} = \dfrac{x(x+2)}{(x+2)^2} = \dfrac{x}{x+2}$,
$x \neq -2$

112. $\dfrac{x^2+6x+9}{x^2-4}\cdot\dfrac{x+3}{x-2}=\dfrac{(x+3)^2}{(x-2)(x+2)}\cdot\dfrac{x+3}{x-2}$

$=\dfrac{(x+3)^3}{(x-2)^2(x+2)}$,

$x\neq 2,-2$

113. $\dfrac{6x+2}{x^2-1}\div\dfrac{3x^2+x}{x-1}$

$=\dfrac{2(3x+1)}{(x-1)(x+1)}\div\dfrac{x(3x+1)}{x-1}$

$=\dfrac{2(3x+1)}{(x-1)(x+1)}\cdot\dfrac{x-1}{x(3x+1)}$

$=\dfrac{2}{x(x+1)}$,

$x\neq 0,1,-1,-\dfrac{1}{3}$

114. $\dfrac{x^2-5x-24}{x^2-x-12}\div\dfrac{x^2-10x+16}{x^2+x-6}$

$=\dfrac{(x-8)(x+3)}{(x-4)(x+3)}\div\dfrac{(x-2)(x-8)}{(x+3)(x-2)}$

$=\dfrac{x-8}{x-4}\cdot\dfrac{x+3}{x-8}$

$=\dfrac{x+3}{x-4}$,

$x\neq -3,4,2,8$

115. $\dfrac{2x-7}{x^2-9}-\dfrac{x-10}{x^2-9}=\dfrac{2x-7-(x-10)}{x^2-9}$

$=\dfrac{x+3}{(x+3)(x-3)}$

$=\dfrac{1}{x-3}$,

$x\neq 3,-3$

116. $\dfrac{3x}{x+2}+\dfrac{x}{x-2}=\dfrac{3x}{x+2}\cdot\dfrac{x-2}{x-2}+\dfrac{x}{x-2}\cdot\dfrac{x+2}{x+2}$

$=\dfrac{3x^2-6x+x^2+2x}{(x+2)(x-2)}$

$=\dfrac{4x^2-4x}{(x+2)(x-2)}$

$=\dfrac{4x(x-1)}{(x+2)(x-2)}$,

$x\neq 2,-2$

117. $\dfrac{x}{x^2-9}+\dfrac{x-1}{x^2-5x+6}$

$=\dfrac{x}{(x-3)(x+3)}+\dfrac{x-1}{(x-2)(x-3)}$

$=\dfrac{x}{(x-3)(x+3)}\cdot\dfrac{x-2}{x-2}+\dfrac{x-1}{(x-2)(x-3)}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{x(x-2)+(x-1)(x+3)}{(x-3)(x+3)(x-2)}$

$=\dfrac{x^2-2x+x^2+2x-3}{(x-3)(x+3)(x-2)}$

$=\dfrac{2x^2-3}{(x-3)(x+3)(x-2)}$

$x\neq 3,-3,2$

118. $\dfrac{4x-1}{2x^2+5x-3}-\dfrac{x+3}{6x^2+x-2}$

$=\dfrac{4x-1}{(2x-1)(x+3)}-\dfrac{x+3}{(2x-1)(3x+2)}$

$=\dfrac{4x-1}{(2x-1)(x+3)}\cdot\dfrac{3x+2}{3x+2}$

$-\dfrac{x+3}{(2x-1)(3x+2)}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{12x^2+8x-3x-2-x^2-6x-9}{(2x-1)(x+3)(3x+2)}$

$=\dfrac{11x^2-x-11}{(2x-1)(x+3)(3x+2)}$,

$x\neq\dfrac{1}{2},-3,-\dfrac{2}{3}$

119. $\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}=\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}\cdot\dfrac{6x}{6x}$

$=\dfrac{6-3x}{2x-x^2}$

$=\dfrac{-3(x-2)}{-x(x-2)}$

$=\dfrac{3}{x}$,

$x\neq 0,2$

120. $$\frac{3+\frac{12}{x}}{1-\frac{16}{x^2}} = \frac{3+\frac{12}{x}}{1-\frac{16}{x^2}} \cdot \frac{x^2}{x^2}$$
$$= \frac{3x^2+12x}{x^2-16}$$
$$= \frac{3x(x+4)}{(x+4)(x-4)}$$
$$= \frac{3x}{x-4},$$
$$x \neq 0, 4, -4$$

121. $$\frac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}} = \frac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}} \cdot \frac{x+3}{x+3}$$
$$= \frac{3(x+3)-1}{3(x+3)+1}$$
$$= \frac{3x+9-1}{3x+9+1}$$
$$= \frac{3x+8}{3x+10},$$
$$x \neq -3, -\frac{10}{3}$$

Chapter P Test

1. $$5(2x^2-6x)-(4x^2-3x) = 10x^2-30x-4x^2+3x$$
$$= 6x^2-27x$$

2. $$7+2[3(x+1)-2(3x-1)]$$
$$= 7+2[3x+3-6x+2]$$
$$= 7+2[-3x+5]$$
$$= 7-6x+10$$
$$= -6x+17$$

3. $$\{1,2,5\} \cap \{5,a\} = \{5\}$$

4. $$\{1,2,5\} \cup \{5,a\} = \{1,2,5,a\}$$

5. $$(2x^2y^3-xy+y^2)-(-4x^2y^3-5xy-y^2)$$
$$= 2x^2y^3-xy+y^2+4x^2y^3+5xy+y^2$$
$$= 2x^2y^3+4x^2y^3-xy+5xy+y^2+y^2$$
$$= 6x^2y^3+4xy+2y^2$$

6. $$\frac{30x^3y^4}{6x^9y^{-4}} = 5x^{3-9}y^{4-(-4)} = 5x^{-6}y^8 = \frac{5y^8}{x^6}$$

7. $$\sqrt{6r}\cdot\sqrt{3r} = \sqrt{18r^2} = \sqrt{9r^2}\cdot\sqrt{2} = 3r\sqrt{2}$$

8. $$4\sqrt{50}-3\sqrt{18} = 4\sqrt{25\cdot 2}-3\sqrt{9\cdot 2}$$
$$= 4\cdot 5\sqrt{2}-3\cdot 3\sqrt{2}$$
$$= 20\sqrt{2}-9\sqrt{2}$$
$$= 11\sqrt{2}$$

9. $$\frac{3}{5+\sqrt{2}} = \frac{3}{5+\sqrt{2}}\cdot\frac{5-\sqrt{2}}{5-\sqrt{2}}$$
$$= \frac{3(5-\sqrt{2})}{25-2}$$
$$= \frac{3(5-\sqrt{2})}{23}$$

10. $$\sqrt[3]{16x^4} = \sqrt[3]{8x^3\cdot 2x}$$
$$= \sqrt[3]{8x^3}\cdot\sqrt[3]{2x}$$
$$= 2x\sqrt[3]{2x}$$

11. $$\frac{x^2+2x-3}{x^2-3x+2} = \frac{(x+3)(x-1)}{(x-2)(x-1)} = \frac{x+3}{x-2},$$
$$x \neq 2, 1$$

12. $$\frac{5\times 10^{-6}}{20\times 10^{-8}} = \frac{5}{20}\cdot\frac{10^{-6}}{10^{-8}} = 0.25\times 10^2 = 2.5\times 10^1$$

13. $$(2x-5)(x^2-4x+3)$$
$$= 2x^3-8x^2+6x-5x^2+20x-15$$
$$= 2x^3-13x^2+26x-15$$

14. $$(5x+3y)^2 = (5x)^2+2(5x)(3y)+(3y)^2$$
$$= 25x^2+30xy+9y^2$$

15. $$\frac{2x+8}{x-3}\div\frac{x^2+5x+4}{x^2-9}$$
$$= \frac{2(x+4)}{x-3}\div\frac{(x+1)(x+4)}{(x-3)(x+3)}$$
$$= \frac{2(x+4)}{x-3}\cdot\frac{(x-3)(x+3)}{(x+1)(x+4)}$$
$$= \frac{2(x+3)}{x+1},$$
$$x \neq 3, -1, -4, -3$$

16. $\dfrac{x}{x+3}+\dfrac{5}{x-3}$

$=\dfrac{x}{x+3}\cdot\dfrac{x-3}{x-3}+\dfrac{5}{x-3}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{x(x-3)+5(x+3)}{(x+3)(x-3)}$

$=\dfrac{x^2-3x+5x+15}{(x+3)(x-3)}$

$=\dfrac{x^2+2x+15}{(x+3)(x-3)}, x\neq 3,-3$

17. $\dfrac{2x+3}{x^2-7x+12}-\dfrac{2}{x-3}$

$=\dfrac{2x+3}{(x-3)(x-4)}-\dfrac{2}{x-3}$

$=\dfrac{2x+3}{(x-3)(x-4)}-\dfrac{2}{x-3}\cdot\dfrac{x-4}{x-4}$

$=\dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\dfrac{2x+3-2x+8}{(x-3)(x-4)}$

$=\dfrac{11}{(x-3)(x-4)},$

$x\neq 3, 4$

18. $\dfrac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}=\dfrac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}\cdot\dfrac{3x}{3x}=\dfrac{3-x}{3},$

$x\neq 0$

19. $x^2-9x+18=(x-3)(x-6)$

20. $x^3+2x^2+3x+6=x^2(x+2)+3(x+2)$

$=(x^2+3)(x+2)$

21. $25x^2-9=(5x)^2-3^2=(5x-3)(5x+3)$

22. $36x^2-84x+49=(6x)^2-2(6x)\cdot 7+7^2$

$=(6x-7)^2$

23. $y^3-125=y^3-5^3=(y-5)(y^2+5y+25)$

24. $(x^2+10x+25)-9y^2$

$=(x+5)^2-9y^2$

$=(x+5-3y)(x+5+3y)$

25. $x(x+3)^{-\frac{3}{5}}+(x+3)^{\frac{2}{5}}$

$=(x+3)^{-\frac{3}{5}}\left[x+(x+3)\right]$

$=(x+3)^{-\frac{3}{5}}(2x+3)=\dfrac{2x+3}{(x+3)^{\frac{3}{5}}}$

26. $-7,-\dfrac{4}{5},0,0.25,\sqrt{4},\dfrac{22}{7}$ are rational numbers.

27. $3(2+5)=3(5+2)$;
commutative property of addition

28. $6(7+4)=6\cdot 7+6\cdot 4$
distributive property of multiplication over addition

29. $0.00076=7.6\times10^{-4}$

30. $27^{-\frac{5}{3}}=\dfrac{1}{27^{\frac{5}{3}}}=\dfrac{1}{\left(\sqrt[3]{27}\right)^5}=\dfrac{1}{(3)^5}=\dfrac{1}{243}$

31. $2\left(6.6\times10^9\right)=13.2\times10^9=1.32\times10^{10}$

32. a. 2003 is 14 years after 1989.

$M=-0.28n+47$

$M=-0.28(14)+47$

$=43.08$

In 2003, 43.08% of bachelor's degrees were awarded to men. This overestimates the actual percent shown by the bar graph by 0.08%.

b. $R=\dfrac{M}{W}=\dfrac{-0.28n+47}{0.28n+53}$

c. $R=\dfrac{-0.28n+47}{0.28n+53}$

$R=\dfrac{-0.28(25)+47}{0.28(25)+53}$

$=\dfrac{2}{3}$

Three women received bachelor's degrees for every two men. This describes the data exactly.